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# Simultaneous Signaling in Elimination Contests

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## Abstract

This paper analyzes the signaling effect of bidding in a two-round elimination contest. Before the final round, bids in the preliminary round are revealed and act as signals of the contestants' private valuations. Depending on his valuation, a contestant may have an incentive to bluff or sandbag in the preliminary round in order to gain an advantage in the final round. I analyze this signaling effect and characterize the equilibrium in this game. Compared to the benchmark model, in which private valuations are revealed automatically before the final round and thus no signaling of bids takes place, I find that strong contestants bluff and weak contestants sandbag. In a separating equilibrium, bids in the preliminary round fully reveal the contestants' private valuations. However, this signaling effect makes the equilibrium bidding strategy in the preliminary round steeper for high valuations and flatter for low valuations compared to the benchmark model.

Key words: all-pay auction, elimination contests, incomplete information, lottery, signaling  
JEL classification: C72, D44, D82

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# 1 Introduction

Contests are frequently used to model R&D races, political elections, science contests, promotions, rent-seeking and lobbying, etc. In a contest, players compete for prizes by submitting “bids” and, regardless of results, all “bids” are sunk. Different situations give different interpretations of a “bid”. For example, in R&D races, political elections, and rent-seeking and lobbying, it represents the amount of money spent by each player, while in science contests and promotions, it becomes the amount of effort exerted by a contestant or a worker. Depending on how a winner is selected, two major branches are distinguishable in the literature. Hurely and Shogren [12, 13], Nti [19], Tullock [22] model contests as lotteries where a player’s winning probability is equal to the ratio of his own bid to the total bids. Bay et. al [2], Hillman and Riley [9], on the other hand, model contests as all-pay auctions where the winner is the one with the highest bid.

In many real life contests, players are initially divided into a few groups and they first compete within their own subgroups and then winners from each group compete again in later stages. As pointed out in Moldovanu and Sela [18]: “Besides sports, elimination contests are very popular and widely used in the following situations: (1) in the organization of internal labor markets in large firms and public agencies, the sub-contests are usually regional or divisional, and the prizes are promotions to well-defined (and usually equally paid) positions on the next rung of the hierarchy-ladder; (2) in political competition (e.g., for the US presidency), candidates first spend resources to secure their party’s nomination, and later, if they are nominated, spend more resources to get elected; and (3) science contests among university or high-school students, e.g., the Mathematics Olympiad.”

In real life elimination contests, contestants’ actions in each round are usually publicly observed before the next round. For example, in the labor market promotions, workers usually know how well their colleagues have done in the past; in many political campaigns, expenditures exerted by candidates in securing votes are also publicly observed. Contestants anticipate that their actions in the earlier round act as signals of their types (such as strength, ability, valuation and talent), and that their signals influence their rivals’ beliefs and strategies in later rounds.<sup>1</sup> Therefore, contestants have incentives either to engage in “bluffing” by submitting a bid higher than their true types would submit in equilibrium or to engage in “sandbagging” by submitting a bid lower than their true types would submit in equilibrium in order to gain some advantages in the later round. I investigate this signaling effect by considering a two-round elimination contest under incomplete information. I assume that the preliminary round is an all-pay auction and that the final round is a lottery.

Since I am interested in examining the signaling effect, I focus on the equilibrium in which the signaling occurs, i.e the separating equilibrium.<sup>2</sup> In a separating equilibrium, players’ actions (bids) in the preliminary round fully reveal their valuations, and there is complete information in the final round. Generally speaking, a bid in the preliminary round has two effects. It changes

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<sup>1</sup>Potentially, a contestant’s type is multi-dimensional. In this paper, I assume that a type is limit to the valuation of the prize.

<sup>2</sup>Although the pooling equilibrium is of interest, most of the related literature focuses on the separating equilibrium (see Goeree [3], Haile [6, 7, 8], and Mailath [15]).

a player’s winning probability in the preliminary round, and it also has a signaling effect. Since I want to investigate how the signaling effect affects a player’s strategy, it is natural to look at the benchmark model in which players’ valuations are revealed automatically and become common knowledge before the final round. This benchmark excludes the signaling effect of a bid while preserving its effect on the winning probability.

Comparing this benchmark with my original model fully characterizes the signaling effect. This paper finds that weak contestants sandbag and strong contestants bluff in the preliminary round in the presence of the signaling effect.<sup>3</sup> In a separating equilibrium, a bid in the preliminary round fully reveals a contestant’s private valuation. However, this signaling effect imposes a downward pressure on the equilibrium bidding strategy of weak contestants and an upward pressure on the equilibrium bidding strategy for strong contestants in the preliminary round. In other words, this signaling effect makes the equilibrium bidding strategy in the preliminary round steeper for high valuations and flatter for low valuations compared to the benchmark. Intuitively, the signaling effect works as follows. Since a player’s expected valuation of entering the final round is a decreasing function of his rival’s bid in the final round, there is an incentive for players to disguise their own valuations in order to reduce their rivals’ bids in the final round. Generally speaking, if a player is strong, he anticipates that he will have a greater chance to meet a player weaker than himself in the final round. As a result, he bluffs in the preliminary round since he wants to discourage his rival in the final round. If he is a weak player, he anticipates that he will have a greater chance to meet a player stronger than himself in the final round. As a result, he sandbags in the preliminary round since he would want his rival to underestimate him in the final round.

In this paper, a separating equilibrium may fail to exist due to weak players’ sandbagging in the preliminary round. This happens when the signaling effect becomes strong enough to dominate the effect that pretending to be weaker decreases the winning probability. In this case, it is always better for a player to under-represent his valuation in the preliminary round, since the gain increases while the bidding cost decreases. I am able to identify a sufficient condition to guarantee the existence of a separating equilibrium.

A salient feature of this model is that the outcome in the preliminary round (all-pay auction) is more sensitive to the bids than that in the final round (lottery). The reason is the tractability. First, if I model both rounds as lotteries, then at the beginning of the preliminary round, all players face symmetric incomplete information. However, there is no tractable solution for lottery under incomplete information in general.<sup>4</sup> Second, if I model both rounds as all-pay auctions, then, as shown in Zhang [23], there exists no separating equilibrium.<sup>5</sup> Third, it is well known in the contest literature that if the winning probability in the lottery is determined by  $\frac{b_k^\lambda}{\sum_{i=1}^I b_i^\lambda}$ , where  $b_i$  is player  $i$ ’s bid and  $\lambda$  is known as the sensitivity of the administrators, then an all-pay auction is equivalent to a lottery when  $\lambda$  goes to infinity. Therefore, I am actually modelling the two rounds with lotteries but with different sensitivity of the administrators.

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<sup>3</sup>Horner and Sahuguet [11] state “bluffing (respectively sandbagging) occurs when a weak (respectively strong) player seeks to deceive his opponent into thinking that he is strong (respectively weak).”

<sup>4</sup>Several papers in the literature, Hurely and Shogren [12, 13], and Malueg and Yates [16], have been written under special settings, such as discrete private information or one-sided asymmetric information.

<sup>5</sup>The paper is available at <http://www.econ.queensu.ca/students/phds/zhangjun/>

Although the main reason for this heterogeneity is the tractability, it is not vacuous in real life elimination contests. In the USA, success in the primaries is more sensitive to effort than success in the presidential election since the latter is prone to more noise. In NHL or NBA tournaments, the regular season have many more games and therefore the teams entering the playoffs are usually the better teams. In contrast, the playoffs have less games and the result is more randomness. Many famous TV shows also fit my model.<sup>6</sup> For example, in “American Idol”, the contestants compete in their own divisions for tickets to Hollywood and the winners compete again in the final round.<sup>7</sup> In the preliminary rounds, it is only the judges’ votes that count. Given that the judges are experts, it is very likely that the result in this round will be very sensitive to effort and, as a result, it is the singers with more talent who win, similar to an all-pay auction. When winners get to Hollywood, the votes of the judges no longer count and only the votes of viewers count. Since viewers are not experts, it is possible that success may not be sensitive to effort in this stage and the final round becomes a lottery.<sup>8</sup>

## 2 Related Literature

Early work on eliminating contests considers the case of complete information (See Groh et al [4], Horen and Riezman [10], Hwang [14], Rosen [20], and Schwenk [21]). However, incomplete information is conventionally an interesting topic in economic theory. Moldovanu and Sela [18] consider a two-round elimination contest under incomplete information, but assume that the finalists only know that their rivals are the winners from other groups. Therefore, there is no information transmission regarding players’ actions in the first round. If all the prizes are awarded to the winner in the final round, the model is equivalent to a static contest with several prizes in terms of revenue and efficiency. They point out in their paper that “an interesting avenue is to focus on the role of information in contests with multiple rounds”, which is the motivation for this paper.

To my knowledge, this is the first paper to talk about the strategic impact of signalling in contests. Lai and Matros (2006) have already considered a two-round elimination contest with full revelation or no revelation of bidders’ bids and characterized the corresponding equilibria. In their paper, both rounds are all-pay auctions. Their model is more general than ours, allowing for more players dividing into more groups in the first round. Furthermore, there are interim prizes for the first round winners. However, their approach is different from ours. They assume that the first-round winners are committed to act according to their pretended type in the second round in a deviation. Furthermore, a deviator’s valuation also becomes that of the pretended type. That is, when a player deviates by pretending to be a different type, he becomes that pretending type in the second round. Wang and Zhang [23] assume that when a player deviates, he remains being his original type. They prove that if players act optimally in both stages, no symmetric separating equilibrium exists. In contrast, under the setup of this paper, in which the preliminary round is

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<sup>6</sup>I am grateful for James Amegashie for alerting me of this example

<sup>7</sup>MY model is a simple version of the “American Idol”. In my model, there is only one winner in the preliminary round; in contrast, in the “American Idol”, there are multiple tickets to go to the Hollywood in the preliminary round

<sup>8</sup>Indeed, in the recent season of “American Idol”, there was a contestant with bad performance, Sanjaya, who the judges did not like must kept advancing in the final round because the viewers liked him.

an all-pay auction and the final round is a lottery, I am able to identify a separating symmetric Perfect Bayesian Nash Equilibrium.

MY paper is also related to Amegashie [1]. He analyzes the signaling effect in dynamic contests with one-sided incomplete information and finds that informed players exert higher effort in the preliminary round when the opponent in the final round is weaker than they are and vice versa. In his model, both of the rounds are lotteries. Players are fooled in equilibrium because of bounded rationality. MY paper investigates a two-sided incomplete information model with fully rational players.

Finally, my paper is also related to the literature on signaling in auctions, in which all players have the chance to signal and bids are made simultaneously. Goeree [3] considers an auction followed by an aftermarket, in which bidders compete for an advantage in future strategic interactions.<sup>9</sup> Haile [6, 7, 8] considers an auction followed by a resale auction organized by the first-round winner. Bidders have noisy private signals and their information further improves in the resale round. In all of those papers, the authors focus on the separating Perfect Bayesian Nash Equilibrium, which is also the equilibrium concept employed in my paper. However, a separating equilibrium may not exist under certain situations.<sup>10</sup> It is worthwhile to mention that Mailath [15] gives a sufficient condition to ensure the existence of a separating equilibrium in simultaneous signaling two-period games. Unfortunately, his result can not be applied to my model directly since he assumes that the payoffs are additively separable, a condition which does not apply in my model. However, the above paper still provides us with helpful insight in the analysis.

The rest of the paper is organized as follows. Section 2 analyzes the models with and without the signaling effect. Section 3 provides a comparison of the two models. Section 4 concludes.

### 3 The model

I consider a two-round elimination contest described as follows. There are  $2N$  risk neutral players in the contest. All players are divided into two groups. Player 1 to  $N$  are in group A while player  $N + 1$  to  $2N$  are in group B. Players first compete within their group in the preliminary round simultaneously. The winner of group A is denoted as player A, and the winner of group B is denoted as player B. The winners from the preliminary round (players A and B) enter the final round and compete for the prize.

Players have private information regarding their own valuation of the prize. Assume player  $i$ 's valuation is  $V_i$ , with realization denoted by  $v_i$ . Although players do not know other players' valuations, they believe that they are drawn independently from a commonly known distribution  $F(\cdot)$ , with associated density function  $f(\cdot)$  and support  $\mathcal{V} = [\underline{v}, \bar{v}]$ . I assume that both the valuation and the density of valuation are bounded and away from 0, i.e.  $0 < \underline{v} < \bar{v} < +\infty$  and  $0 < f(v) <$

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<sup>9</sup>Also, in the introduction, he provides a good review of the literature on the signaling in auctions.

<sup>10</sup>Goeree [3] states "if bidders want to understate their private information, a separating equilibrium may fail to exist when the incentives to signal via a lower bid are stronger for higher valuations". Meanwhile, Haile [8] also finds that, due to the signaling effect, player's objective function is not quasiconcave, and a separating equilibrium may fail to exist.

$+\infty$  for  $v \in [\underline{v}, \bar{v}]$ .

Player  $i$  competes with his rivals by making a bid  $b_i$  and all bids are submitted simultaneously. Regardless of success, all players pay for their bids. Therefore, a player's payoff is equal to his valuation multiplied by the winning probability less his bid. In the preliminary round the competition technology is an all pay auction: the player with the highest effort wins. In contrast, in the final round the competition technology is a lottery: a player's winning probability is equal to the ratio of his own bid to the total bid.

### 3.1 The benchmark

I assume that regardless of the outcome in the preliminary round, players' valuations become common knowledge after the preliminary round and before the final round. Though it is not clear how this mechanism is to be implemented, similar mechanisms have been used to analyze signaling in auctions with an aftermarket [3], auctions with resale [5], and collusion in auctions [17]. Here, the benchmark model is presented without worrying about the signaling effect, which will be analyzed later on.

I first describe the timing of the game:

1.  $2N$  players are equally divided into 2 groups: group A and group B.
2. Players privately learn their valuations.
3. A preliminary round contest is held in each group, using an all-pay auction.
4. After the preliminary round and before the final round, players' valuations become common knowledge.
5. The winners from the preliminary round, players A and B, compete in the final round, which is held using a lottery.

To solve the model, I employ the concept of Perfect Bayesian Nash Equilibrium (PBNE). A PBNE is a pair of strategies for each player and a posterior belief distribution, where the strategy profile is sequentially rational given the belief system and the belief is derived from the strategy profile through Bayes' rule whenever possible. Since all payers are *ex-ante* identical before learning their valuations, I look for the symmetric preliminary round bidding strategy. Within the equilibria, I focus on the separating equilibrium that, in the preliminary round, players bid according to a strictly increasing function. I first look at the final round.

#### Final round strategies

The winners from the preliminary round, players A and B, enter the final round. Since all players' valuations are revealed before the final round, players' beliefs about one another's valuations are not affected by preliminary round actions. As a result, the final round is a complete information game, where players simultaneously choose their bids. Throughout this paper, superscripts denote

the round, P or F, and the subscripts denote the player. The winning probability is given by a simple form:

$$Prob\{player\ i\ wins\} = \frac{b_i^F}{b_i^F + b_j^F} \quad i, j \in \{A, B\}, i \neq j$$

Player  $i$ 's problem in the final round is given by:

$$\max_{b_i^F} v_i \frac{b_i^F}{b_i^F + b_j^F} - b_i^F$$

It is useful to note that a player's payoff is a decreasing function of his rival's bid. The following lemma gives the equilibrium.

**Lemma 1** *In the complete information contest played by two players with valuations  $v_i$  and  $v_j$ , in which players choose bids simultaneously and the winning probability is equal to the ratio of a player's own bid to the total bids, in equilibrium, players choose their bids as follows.*

$$b_i^F(v_A, v_B) = \frac{v_i^2 v_j}{(v_i + v_j)^2} \quad \forall i, j \in \{A, B\}, i \neq j$$

With the associated payoff respectively given by:

$$\Pi_i^F(v_A, v_B) = \frac{v_i^3}{(v_i + v_j)^2} \quad \forall i, j \in \{A, B\}, i \neq j$$

I refer to Nti [19] for the details of the proof.

From Lemma 1, I know that a player's bid is an increasing function of his rival's valuation when the rival's valuation is lower than his valuation, and a decreasing function of his rival's valuation when the rival's valuation is higher than his valuation.<sup>11</sup> This lemma is important for understanding players' strategies when they have the opportunity to signal.

### Preliminary round strategies

Having solved the strategy in the final round, I move on to the preliminary round. I assume that all players adopt the same strictly increasing bidding strategy  $b^P(\cdot)$ . In equilibrium, a player with the lower bound valuation bids zero since he has a zero probability of winning. A player will not bid more than  $b^P(\bar{v})$  since bidding  $b^P(\bar{v})$  gives him the same winning probability while saving the cost. Therefore, choosing a bid to maximize one's payoff is equivalent to report one's type optimally. Since players are *ex-ante* identical, I choose player 1 as a representative. Given that all other players adopt  $b^P(\cdot)$ , player 1's problem at the beginning of the preliminary round is given as follows:

$$\Pi_1^P(v_1) = \max_w E\{\Pi_1^F(v_1, V_B)\mathcal{I}_{\{w > V_i, \forall i=2, \dots, N\}}|v_1\} - b^P(w)$$

where  $\mathcal{I}_{\{\cdot\}}$  is an indicator function and throughout the paper all the expectations are taken on

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<sup>11</sup>Formally,  $\frac{\partial b_i^F(v_A, v_B)}{\partial v_j} = \frac{v_i^2(v_i - v_j)}{(v_i + v_j)^3}$ . When  $v_j < v_i$ ,  $\frac{\partial b_i^F(v_A, v_B)}{\partial v_j} > 0$ ; when  $v_j > v_i$ ,  $\frac{\partial b_i^F(v_A, v_B)}{\partial v_j} < 0$ .



random variables, i.e the upper case letters.

The first term is the expected gain from bidding and the second term is the cost of bidding. If player 1 loses in the preliminary round, he gains nothing; if he wins the preliminary round, he gains if he also wins the final round and the payoff in the final round is given by  $\Pi_1^F(v_1, v_B) = \frac{v_1^3}{(v_1+v_B)^2}$  from Lemma 1. Meanwhile, player 1 knows that player B is the winner of group B. Since the preliminary round is an all-pay auction, the winner must be the one with the highest valuation in group B. For instance, player 1, if he can enter the final round, believes that player B's valuation is the first order statistic among all the players in group B, i.e. with *cdf*  $F(v_B)^N$ . Hence, player 1's problem turns out to be:

$$\begin{aligned}\Pi_1^P(v_1) &= \max_w E\left\{\frac{v_1^3}{(v_1 + V_B)^2} \mathcal{I}_{\{w > V_i, \forall i=2, \dots, N\}} | v_1\right\} - b^P(w) \\ &= \max_w E\left\{\mathcal{I}_{\{w > V_i, \forall i=2, \dots, N\}}\right\} E\left\{\frac{v_1^3}{(v_1 + V_B)^2} | v_1\right\} - b^P(w) \\ &= \max_w F(w)^{N-1} \int_{\underline{v}}^{\bar{v}} \frac{v_1^3}{(v_1 + v_B)^2} dF(v_B)^N - b^P(w)\end{aligned}$$

I can interpret this function as follows. The term  $F(w)^{N-1}$  is the winning probability in the preliminary round, the term  $\int_{\underline{v}}^{\bar{v}} \frac{v_1^3}{(v_1+v_B)^2} dF(v_B)^N$  is the expected valuation of entering the final round, and the term  $b^P(w)$  is his bid. Note that the payoff from losing is zero in the model setup. The following proposition gives the equilibrium bidding strategy in the preliminary round and a summary of the equilibrium of the whole game.

**Proposition 1** *In the elimination contest excluding the signaling effect, in which players' valuations are automatically revealed after the preliminary round and before the final round, the separating symmetric PBNE is as follows.*

*In the preliminary round, all players bid according to a strictly increasing bidding function:*

$$b_{nosignaling}^P(v) = \int_{\underline{v}}^v \int_{\underline{v}}^{\bar{v}} \frac{\xi^3}{(\xi + \zeta)^2} dF(\zeta)^N dF(\xi)^{N-1}$$

*In the final round, the winners from the preliminary round, players A and B, knowing each other's valuation, bid as described by Lemma 1.*

**Proof:** see the appendix

The expected valuation of entering the final round just depends on a player's own valuation and the distribution of valuation. A high valuation player evaluates the final round higher than a low valuation player does. So if I redefine the valuation as the expected valuation of entering the final round, then the preliminary round is the same as a normal form all-pay auction. The equilibrium exists under any distribution of valuation. It is easy to check that the bidding function is indeed strictly increasing, which is consistent with my presumption.

Under this setting, preliminary round bids only affect the winning probability in the preliminary round. In contrast, as I can see below, in the game with signaling, action in the preliminary round has an extra effect. There is an inferential impact via the other players' final round strategies, since they infer the player's valuation from his action.

### 3.2 Incorporating signaling effects

I need to replace time 4 in the previous model which excludes the signaling effect:

4. After the preliminary round and before the final round begins, all bids in the preliminary round are observed by all the players and become common knowledge.

Types are not revealed automatically; instead, players' bids in the preliminary round are revealed. If I assume the equilibrium is separating and all players bid according to a strictly increasing function in the preliminary round, say  $b^P(\cdot)$ , then after the bids are revealed players can infer other players' valuations by inverting the bidding function. The final round is a complete information game and coincides with the final round in the model without signaling effects. However, since the finalists are informed of the bids in the preliminary round, and valuations remain unobservable to them, it is conceivable that a player may want to disguise himself by over-representing or under-representing his valuation in the preliminary round in order to gain some advantages in the final round. To solve the model, I employ the concept of PBNE and work backwards.

#### Final round strategies

Suppose that in the preliminary round all players adopt the same strictly increasing bidding function,  $b^P(\cdot)$ . I denote the image of the preliminary round strategy as  $b^P(\mathcal{V}) = [b^P(\underline{v}), b^P(\bar{v})]$ . It is not hard to see that the lower bound of  $b^P(\mathcal{V})$  is  $b^P(\underline{v}) = 0$ , since the player with the lowest valuation has zero probability of entering the final round and thus would not bid more than zero. If a player bids  $b \in b^P(\mathcal{V})$  in the preliminary round, then according to Bayes' rule, other players in the final round believe that his valuation is  $(b^P)^{-1}(b)$ . If a player's bid is outside the image of the preliminary round bidding function, i.e.  $b > b^P(\bar{v})$ ,<sup>12</sup> I assume that all other players believe that he has valuation  $\bar{v}$ . As a result, bidding  $b^P(\bar{v})$  strictly dominates bidding  $b > b^P(\bar{v})$ , since there is no benefit for the higher bid while the cost is higher. Under this specification of the off-path beliefs, no player has any incentive to deviate to a valuation outside the support of the valuation space  $\mathcal{V}$ . Therefore, I will focus on deviations of bids within the image of the preliminary round bidding function.

In the non-deviated continuation game, where all players follow the equilibrium strategy, the final round becomes a lottery with complete information and coincides with the model without the signaling effect. In order to characterize the equilibrium bidding strategies in the preliminary round, I need to examine one more deviated continuation game in the final round. In this continuation game, only one player deviates and does not following his equilibrium bidding function in the preliminary round, but is able to enter the final round.

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<sup>12</sup>Since players can not bid a negative amount, the only possible bid, which is outside the image of the preliminary round bidding function, is  $b > b^P(\bar{v})$ .

Since all players are *ex-ante* symmetric, I choose player 1 as the representative player and assume that he is the one who deviates. Let  $w = (b^P)^{-1}(b) \in \mathcal{V}$  be the valuation other players believe player 1 has, which may or may not be player 1's true valuation. If he loses in the preliminary round, then it does not affect the final round contest. The following analysis applies when he wins and enters the final round, and thus becomes player A.

In the final round, player B, the winner from group B, infers that player A's valuation is  $w$ . Player A learns that player B's valuation is  $v_B$  from his preliminary round bid. Furthermore, player A knows that player B believes that he has valuation  $w$ . Of course, player A knows that his own valuation is actually  $v_A$ . The following lemma describes the equilibrium in this special deviated continuation game.

**Lemma 2** *In the special deviated continuation game in the final round described above, player B bids:*

$$b_B^F(w, v_B) = \frac{v_B^2 w}{(w + v_B)^2}$$

and player A bids:

$$b_A^F(w, v_A, v_B) = \begin{cases} 0 & \text{if } w \geq v_A \text{ and } v_B \geq \frac{w\sqrt{v_A}}{\sqrt{w} - \sqrt{v_A}} \\ \frac{v_B\sqrt{wv_A}}{w+v_B} - \frac{v_B^2 w}{(w+v_B)^2} & \text{otherwise} \end{cases}$$

and player A's associated payoff is:

$$\Pi_A^F(w, v_A, v_B) = \begin{cases} 0 & \text{if } w \geq v_A \text{ and } v_B \geq \frac{w\sqrt{v_A}}{\sqrt{w} - \sqrt{v_A}} \\ (\sqrt{v_A} - \frac{v_B\sqrt{w}}{w+v_B})^2 & \text{otherwise} \end{cases}$$

**Proof:** See appendix

When player A bluffs in the preliminary round and meets a very strong rival in the final round, I may have a corner solution; it is optimal for him to drop out in the competition and bid zero. Otherwise, his bid in the final round is an interior solution.

### Preliminary Round Strategies

I now consider the preliminary round bidding function. Lemma 2 gives players' strategies in the special deviated continuation game when player 1 deviates and enters the final round. Obviously, player 1's expected surplus from entering the final round depends on both his true valuation and his pretended valuation. Suppose that in the preliminary round all other players adopt the equilibrium bidding function  $b^P(\cdot)$  and player 1 has valuation  $v_1$  but pretends to have valuation  $w$ . Then player 1's payoff in the whole game is given by:

$$E\{\Pi_1^F(w, v_1, V_B)\mathcal{I}_{\{w > V_i, \forall i=2, \dots, N\}}|v_1\} - b^P(w)$$

$$= E\{\mathcal{I}_{\{w > V_i, \forall i=2, \dots, N\}}\} E\{\Pi_1^F(w, v_1, V_B)|v_1\} - b^P(w)$$

The first term  $E\{\mathcal{I}_{\{w > V_i, \forall i=2, \dots, N\}}\}$  is the winning probability in the preliminary round, the second term  $E\{\Pi_1^F(w, v_1, V_B)|v_1\}$  is the expected valuation of entering the final round, and the last term is the bid.

If he chooses to under-represent his valuation ( $w \leq v_1$ ) in the preliminary round, then his problem becomes:

$$\max_w F(w)^{N-1} \int_{\underline{v}}^{\bar{v}} \left(\sqrt{v_1} - \frac{v_B \sqrt{w}}{w + v_B}\right)^2 dF(v_B)^N - b^P(w)$$

If he chooses to over-represent his valuation in the preliminary round, ( $w \geq v_1$ ), then his problem becomes:

$$\max_w F(w)^{N-1} \int_{\underline{v}}^{\min\left\{\bar{v}, \frac{w\sqrt{v_1}}{\sqrt{w}-\sqrt{v_1}}\right\}} \left(\sqrt{v_1} - \frac{v_B \sqrt{w}}{w + v_B}\right)^2 dF(v_B)^N - b^P(w)$$

However, if he just over-represents his valuation locally, i.e. by an infinitely small amount, then his problem becomes:

$$\max_w F(w)^{N-1} \int_{\underline{v}}^{\bar{v}} \left(\sqrt{v_1} - \frac{v_B \sqrt{w}}{w + v_B}\right)^2 dF(v_B)^N - b^P(w)$$

A weak player anticipates that if he can enter the final round he has a greater chance to meet a player stronger than him. In that case, he would like to sandbag in the preliminary round and induce his rival in the final round to underestimate him and to bid less. If this signaling effect dominates the effect of “sandbagging” decreasing the winning probability, then it is always better to sandbag in the preliminary round and truthful reporting would not be the optimal choice. As a result, a separating equilibrium may not exist. I need some restrictions on the distribution of valuation to exclude this situation.

**Assumption 1** Let  $G(w, v_1)$  be defined by  $F(w)^{N-1} \int_{\underline{v}}^{\bar{v}} \left(\sqrt{v_1} - \frac{v_B \sqrt{w}}{w + v_B}\right)^2 dF(v_B)^N$ . I assume that:

- (i)  $G_1(v_1, v_1) > 0, \forall v_1 \in (\underline{v}, \bar{v}]$
- (ii)  $G_{12}(w, v_1) > 0, \forall w, v_1 \in (\underline{v}, \bar{v}]$

As in many dynamic models with asymmetric information, necessary and sufficient conditions that ensure existence are difficult to identify. Here, I give only one sufficient condition. Restriction (i) ensures that  $b^P(\cdot)$  is strictly increasing, and restriction (ii) is the single-crossing condition.<sup>13</sup>

The restrictions (i) and (ii) given in the above assumption seem to be complicated and people may doubt the existence of such a distribution of valuation that can satisfy both restrictions. The following lemma is a sufficient condition to ensure that the distribution of valuation satisfies the

<sup>13</sup>A more straightforward formulation of the single crossing condition is as follows. Define  $U(v_1, w, b^P) = G(w, v_1) - b^P$ , where  $v_1$  is player's true valuation and  $w$  is his perceived valuation when he bids  $b^P$ . Single crossing condition implies that:  $U_{b^P}(v_1, w, b^P)/U_w(v_1, w, b^P) = \frac{-1}{G_1(w, v_1)}$  is increasing in  $v_1$ , i.e. the ratio of the marginal cost of signaling to the marginal benefit of signaling is lower for higher valuations.

restrictions above. It is straightforward since it just depends on the minimum valuation  $\underline{v}$ , the ratio of the maximum valuation to the minimum valuation  $R = \frac{\bar{v}}{\underline{v}}$ , and the minimum of the density function  $M = \min_v f(v)$ .

**Lemma 3** *The restrictions (i) and (ii) in Assumption 1 are satisfied if the distribution of valuation satisfies the following conditions:*

- $1 < R < \sqrt[3]{4}$
- $(N - 1)M\underline{v} > \max\left\{\frac{(R-1)R^4}{2}, \frac{R^2-R}{8-4R^{\frac{3}{2}}}\right\}$

**Proof:** see appendix

These conditions are most likely to be valid in the political campaign. The value of winning the campaign is usually quite large for any party, and their valuations would not differ too much.

The following proposition gives the equilibrium bidding strategy in the preliminary round and a summary of the equilibrium of the whole game, under Assumption 1.

**Proposition 2** *Suppose Assumption 1 holds, then in the elimination contest with signaling effect, in which all players' bids are revealed after the preliminary round and before the final round, the separating symmetric PBNE is as follows.*

*In the preliminary round, players bid according to a strictly increasing bidding function:*

$$b_{\text{signaling}}^P(v) = \int_{\underline{v}}^v \int_{\underline{v}}^{\bar{v}} \frac{\xi^3}{(\xi+\zeta)^2} dF(\zeta)^N dF(\xi)^{N-1} \\ + \int_{\underline{v}}^v \int_{\underline{v}}^{\bar{v}} \frac{\xi\zeta(\xi-\zeta)F(\xi)^{N-1}}{(\xi+\zeta)^3} dF(\zeta)^N d\xi$$

*In the final round, the winners from the preliminary round, players A and B, knowing each other's valuation by inverting the bidding function, bid as described in Lemma 1.*

**Proof:** See appendix

As I can see, the signaling effect affects the bidding strategy in the preliminary round. Since the equilibrium is a separating one, players can correctly infer their rival's valuation from his bid in the preliminary round in the equilibrium. Therefore, the final round turns into a complete information game and the strategy coincides with that in the model excluding the signaling effect.

## 4 Comparison

As I can see, the strategies in the final round are exactly the same under two different settings. Though the preliminary round strategies are different due to the signaling effect, I can see the

relationship between the two. Recall that:

$$b_{nosignaling}^P{}'(v) = (N-1)F(v)^{N-2}f(v) \int_{\underline{v}}^{\bar{v}} \frac{v^3}{(v+\zeta)^2} dF(\zeta)^N \quad (1)$$

$$\begin{aligned} b_{signaling}^P{}'(v) &= (N-1)F(v)^{N-2}f(v) \int_{\underline{v}}^{\bar{v}} \frac{v^3}{(v+\zeta)^2} dF(\zeta)^N \\ &\quad + F(v)^{N-1} \int_{\underline{v}}^{\bar{v}} \frac{v\zeta(v-\zeta)}{(v+\zeta)^3} dF(\zeta)^N \end{aligned} \quad (2)$$

I call  $b^P{}'(v)$  the marginal willingness to bid. As I can see, the two equations share the same item  $(N-1)F(v)^{N-2}f(v) \int_{\underline{v}}^{\bar{v}} \frac{v^3}{(v+\zeta)^2} dF(\zeta)^N$ , which I call the winning probability effect; while (2) has an extra item:  $S(v) = F(v)^{N-1} \int_{\underline{v}}^{\bar{v}} \frac{v\zeta(v-\zeta)}{(v+\zeta)^3} dF(\zeta)^N$ , which I call the signaling effect. In the model excluding the signaling effect, actions in the preliminary round change the result through the winning probability, which is an increasing function of the bid. In contrast, in the model with the signaling effect, actions in the preliminary round have one additional effect: signaling effect, which is fully characterized by  $S(v)$ .

How the signaling effect affects a player's payoff depends on his valuation. If  $S(v)$  is positive, then it means that pretending to be stronger is good for the player overall. In contrast, if  $S(v)$  is negative, it means that pretending to be weaker is good for the player overall. The specification of the function  $S(v)$  leads to the following crucial result.

**Proposition 3** *If  $R < 2$ , then there exists a threshold  $v^*$ ,  $\underline{v} < v^* < \bar{v}$ , such that for a player with valuation  $\underline{v} < v < v^*$ , his marginal willingness to bid is lower in the presence of the signaling effect (for valuation  $v = \underline{v}$ , the marginal willingness to bid is the same); meanwhile, for a player with valuation  $v^* < v \leq \bar{v}$ , his marginal willingness to bid is higher in the presence of the signaling effect.*

**Proof:** See appendix

See Figure 1 for an illustration.

Although Proposition 3 is for the case  $R < 2$ , I conjecture that the result holds for any distribution of valuation. Given that the distribution of valuation satisfies the restrictions in Assumption 1, the result holds.

This result is intuitive. Since a player's payoff in the final round is a decreasing function of his rival's bid, he has the incentive to disguise himself to induce the rival to bid less in the final round. Under-representing his valuation in the preliminary round makes the rival in the final round underestimate him. This has two effects. On the one hand, it makes the rival stronger than him bid less, which increases his payoff in the final round. On the other hand, it makes the rival weaker than him bid more, which decreases his payoff in the final round. Conversely, over-representing his valuation in the preliminary makes the rival in the final round overestimate him. This has two effects as well. It makes the rival stronger than him bid more, which decreases his payoff in the final round. But it also makes the rival weaker than him bid less, which decreases his payoff in the final round. In other words, if the player is strong, he anticipates that he will have a greater chance to

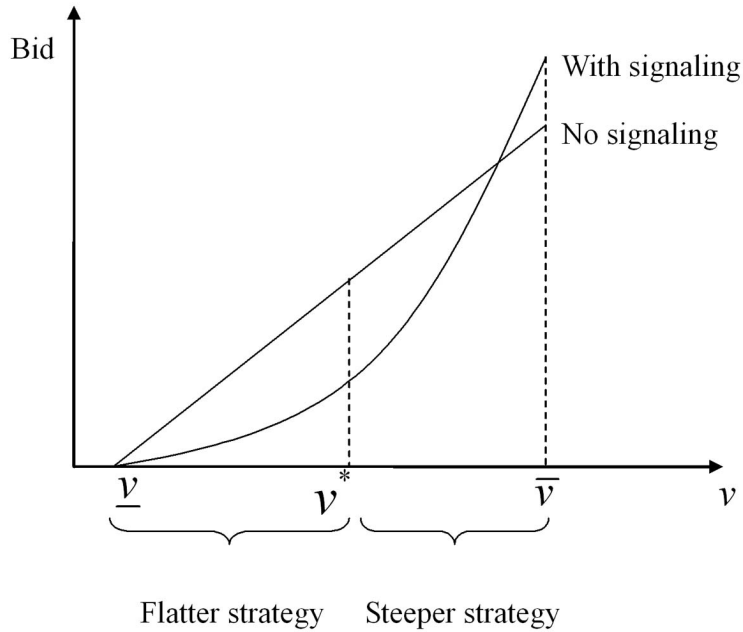


Figure 1: Bidding function with and without the signaling effect

meet a player weaker than himself in the final round, and as a result, he is willing to over-represent his valuation in the preliminary round since he wants to discourage the rival; if he is a weak player, he anticipates that he will have a greater chance to meet a player stronger than himself in the final round, and as a result, he is willing to under-represent his valuation in the preliminary round since he wants them to underestimate him.

For a strong player who has the incentive to over-report his valuation, he may end up with a lower bid compared to the no signaling case, since weak players lower their bids too much.

In the equilibrium under both settings, the winners in the preliminary are the ones with the highest valuations in their own groups, while the final rounds coincide. Furthermore, the expected payoff of a player with lower bound valuation is always zero. If I define the organizer's revenue as the total expected bids, the well-known revenue equivalence theorem would suggest that both models should generate the same revenue. In fact, revenue equivalence does not hold in my model. This is simply because the model without the signaling effect is not a feasible mechanism.

## 5 Conclusion

This paper examines how the signaling effect works in a two-round elimination contest. Players are assumed to *ex-ante* identical and are randomly divided into two groups. In the preliminary round, players compete within their groups and the winners enter the final round. In the benchmark model, players' valuations are automatically revealed in the final round. Thus, the expected valuation of entering the final round just depends on a player's own valuation and the distribution of valuation. In contrast, in the second model, players' valuations are not revealed in the final round while all players' bids in the preliminary round are revealed. Given that in the preliminary round players bid according to a strictly increasing function (separating equilibrium), actions in the preliminary round fully reveal players' valuations and thus affect players' actions in the final round. Since valuations are not known, players have the incentive to disguise themselves. As shown in this paper, weak players are willing to pretend to be weaker and strong players are willing to pretend to be stronger in the presence of the signaling effect, which imposes a downward pressure on the equilibrium bidding strategy for weak players and an upward pressure for the strong players.



## 6 Appendix

The following theorem will be used several times in the proof. A more general version of the lemma can be found in Guesnerie and Laffont (1984). However, special cases were used in several papers, such as McAfee et al (1993) and Myerson (1981).

**Theorem 1** *Consider a general all-pay auction*

$$\max_v U(v, v_1) = \max_v G(v, v_1) - b(v) \quad (3)$$

with boundary condition  $b(\underline{v}) = 0$ . The expected gain function  $G(v, v_1): [\underline{v}, \bar{v}] \times [\underline{v}, \bar{v}] \rightarrow \mathcal{R}$  is twice continuously differentiable and depends on both the true valuation  $v_1$  and the reported valuation  $v$ . Assume that

- $G_1(v_1, v_1) > 0 \forall v_1 \in (\underline{v}, \bar{v}]$ ,
- $G_{12}(v, v_1) > 0 \forall v, v_1 \in (\underline{v}, \bar{v}]$ .

Then  $b(v_1) = \int_{\underline{v}}^{v_1} G_1(\xi, \xi) d\xi$  is the equilibrium bidding function. This bidding function is strictly increasing and fully valuation revealing.

**Proof:**

Suppose there exists a strictly increasing function  $b(v)$  such that truthfully reporting ( $v = v_1$ ) is optimal, then incentive compatibility implies that

$$G_1(v_1, v_1) - b'(v_1) = 0 \Rightarrow b'(v_1) = G_1(v_1, v_1) \Rightarrow b(v_1) = \int_{\underline{v}}^{v_1} G_1(\xi, \xi) d\xi \quad (4)$$

Note that  $b'(v_1) = G_1(v_1, v_1) > 0, \forall v_1 \in (\underline{v}, \bar{v}]$ . So it is true that  $b(v_1)$  is strictly increasing. Given  $b(v_1) = \int_{\underline{v}}^{v_1} G_1(\xi, \xi) d\xi$ , I have

$$\frac{dU(v, v_1)}{dv} = G_1(v, v_1) - b'(v) = G_1(v, v_1) - G_1(v, v) \quad (5)$$

Since  $G_{12}(v, v_1) > 0 \forall v, v_1 \in (\underline{v}, \bar{v}]$ , then  $G_1(v, v_1)$  is a strictly increasing function of  $v_1$ . Hence,  $\frac{dU(v, v_1)}{dv} = G_1(v, v_1) - G_1(v, v) > 0$  if  $v_1 > v$ ; and  $\frac{dU(v, v_1)}{dv} = G_1(v, v_1) - G_1(v, v) < 0$  if  $v_1 < v$ . Thus,  $v = v_1$  is optimal.

**Q.E.D**

**Proof of Lemma 2**

In the final round, player B infers that player A's valuation is  $w$ , and chooses his bid in the final round accordingly, i.e.  $b_B^F(w, v_B) = \frac{v_B^2 w}{(w + v_B)^2}$ . Knowing player B's valuation and his response function  $b_B^F(w, v_B)$ , as well as his own true valuation  $v_A$ , player A responds optimally:

$$\max_{b_A^F} v_A \frac{b_A^F}{b_A^F + b_B^F} - b_A^F$$

where  $b_B^F = \frac{v_B^2 w}{(w+v_B)^2}$ .

The FOC gives us:

$$b_A^F = \max\{\sqrt{v_A b_B^F} - b_B^F, 0\} = \max\left\{\frac{v_B \sqrt{w v_A}}{w + v_B} - \frac{v_B^2 w}{(w + v_B)^2}, 0\right\}$$

I can summarize it as follows:

$$b_A^F(w, v_A, v_B) = \begin{cases} \frac{v_B \sqrt{w v_A}}{w + v_B} - \frac{v_B^2 w}{(w + v_B)^2} & \text{if } w > v_A \text{ and } v_B \leq \frac{w \sqrt{v_A}}{\sqrt{w} - \sqrt{v_A}} \\ \frac{v_B \sqrt{w v_A}}{w + v_B} - \frac{v_B^2 w}{(w + v_B)^2} & \text{if } w \leq v_A \\ 0 & \text{otherwise} \end{cases}$$

Plugging  $b_A^F(w, v_A, v_B)$  into the payoff function, I get the expected payoff:

$$\Pi_A^F(w, v_A, v_B) = \begin{cases} (\sqrt{v_A} - \frac{v_B \sqrt{w}}{w + v_B})^2 & \text{if } w > v_1 \text{ and } v_B \leq \frac{w \sqrt{v_A}}{\sqrt{w} - \sqrt{v_A}} \\ (\sqrt{v_A} - \frac{v_B \sqrt{w}}{w + v_B})^2 & \text{if } w \leq v_A \\ 0 & \text{otherwise} \end{cases}$$

**Q.E.D**

### Proof of Lemma 3

Throughout the proof, recall that I have:

- $1 < R < \sqrt[3]{4}$
- $(N - 1)M\underline{v} > \max\left\{\frac{(R-1)R^4}{2}, \frac{R^2 - R}{8 - 4R^{\frac{3}{2}}}\right\}$

Here  $G(w, v_1) = F(w)^{N-1} \int_{\underline{v}}^{\bar{v}} (\sqrt{v_1} - \frac{v_B \sqrt{w}}{w + v_B})^2 dF(v_B)^N$ .

$G_1(v_1, v_1) > 0, \forall v_1 \in (\underline{v}, \bar{v}]$

$$\begin{aligned} \Rightarrow & (N - 1)F(v_1)^{N-2} f(v_1) \int_{\underline{v}}^{\bar{v}} \frac{v_1^3}{(v_1 + v_B)^2} dF(v_B)^N \\ & + F(v_1)^{N-1} \int_{\underline{v}}^{\bar{v}} \frac{v_1 v_B (v_1 - v_B)}{(v_1 + v_B)^3} dF(v_B)^N > 0, \forall v_1 \in (\underline{v}, \bar{v}] \end{aligned}$$

(6)

$$\begin{aligned} \Rightarrow & (N - 1)f(v_1) \int_{\underline{v}}^{\bar{v}} \frac{v_1^3}{(v_1 + v_B)^2} dF(v_B)^N \\ & + F(v_1) \int_{\underline{v}}^{\bar{v}} \frac{v_1 v_B (v_1 - v_B)}{(v_1 + v_B)^3} dF(v_B)^N > 0, \forall v_1 \in (\underline{v}, \bar{v}] \end{aligned}$$

$G_{12}(w, v_1) > 0, \forall w, v_1 \in (\underline{v}, \bar{v}]$

$$\begin{aligned}
&\Rightarrow (N-1)F(w)^{N-2}f(w) \int_{\underline{v}}^{\bar{v}} \left(1 - \frac{v_B}{w+v_B} \sqrt{\frac{w}{v_1}}\right) dF(v_B)^N \\
&\quad + F(w)^{N-1} \int_{\underline{v}}^{\bar{v}} \frac{v_B(w-v_B)}{2\sqrt{wv_1}(w+v_B)^2} dF(v_B)^N > 0, \forall w, v_1 \in (\underline{v}, \bar{v}] \\
&\Rightarrow (N-1)f(w) \int_{\underline{v}}^{\bar{v}} \left(1 - \frac{v_B}{w+v_B} \sqrt{\frac{w}{v_1}}\right) dF(v_B)^N \\
&\quad + F(w) \int_{\underline{v}}^{\bar{v}} \frac{v_B(w-v_B)}{2\sqrt{wv_1}(w+v_B)^2} dF(v_B)^N > 0, \forall w, v_1 \in (\underline{v}, \bar{v}]
\end{aligned} \tag{7}$$

The idea is very simple. If I can prove that the items inside the integration are always greater than zero under the conditions, then I am done.

$$\begin{aligned}
&(N-1)f(v_1) \frac{v_1^3}{(v_1+v_B)^2} + F(v_1) \frac{v_1 v_B (v_1 - v_B)}{(v_1+v_B)^3} \\
&\geq (N-1)M \frac{v^3}{(\bar{v}+v)^2} + 1 * \frac{\bar{v}v(v-\bar{v})}{(v+\bar{v})^3} \\
&= \frac{(N-1)Mv}{4R^2} + \frac{1}{8}(R^2 - R^3) \\
&= \frac{(N-1)Mv - \frac{(R-1)R^4}{2}}{4R^2} > 0
\end{aligned}$$

Thus, restriction (6) is satisfied.

$$\begin{aligned}
&(N-1)f(w) \left(1 - \frac{v_B}{w+v_B} \sqrt{\frac{w}{v_1}}\right) + F(w) \frac{v_B(w-v_B)}{2\sqrt{wv_1}(w+v_B)^2} \\
&\geq (N-1)f(w) \left(1 - \frac{\bar{v}}{v+\bar{v}} \sqrt{\frac{\bar{v}}{v}}\right) + 1 * \frac{\bar{v}v(v-\bar{v})}{2\sqrt{\bar{v}v}(v+\bar{v})^2} \\
&= (N-1)f(w) \left(1 - \frac{1}{2}R^{\frac{3}{2}}\right) + \frac{1}{8v}(R - R^2) \quad \text{recall that } 1 - \frac{1}{2}R^{\frac{3}{2}} > 0 \\
&\geq (N-1)M \left(1 - \frac{1}{2}R^{\frac{3}{2}}\right) + \frac{1}{8v}(R - R^2) \\
&= \frac{1 - \frac{1}{2}R^{\frac{3}{2}}}{v} \left( (N-1)Mv - \frac{R^2 - R}{8 - 4R^{\frac{3}{2}}} \right) > 0
\end{aligned}$$

Thus, restriction (7) is satisfied.

**Q.E.D**

## Proof for Proposition 2

I first consider the following all-pay auction problem.

$$\max_w F(w)^{N-1} \int_{\underline{v}}^{\bar{v}} \left(\sqrt{v_1} - \frac{v_B \sqrt{w}}{w+v_B}\right)^2 dF(v_B)^N - b^P(w) \tag{8}$$

Since  $G_1(v_1, v_1) > 0$  and  $G_{12}(w, v_1) > 0$ , Theorem 1 tells us that the equilibrium bidding function is:

$$\begin{aligned} b^P(v_1) &= \int_{\underline{v}}^{v_1} \left( \int_{\underline{v}}^{\bar{v}} \frac{\xi^3}{(\xi+v_B)^2} dF(v_B)^N \right) dF(\xi)^{N-1} \\ &+ \int_{\underline{v}}^{v_1} \left( F(\xi)^{N-1} \int_{\underline{v}}^{\bar{v}} \frac{\xi v_B (\xi - v_B)}{(\xi + v_B)^3} dF(v_B)^N \right) d\xi \end{aligned} \quad (9)$$

Now, I move on to prove that under the restrictions above, the bidding strategy (9) consists of an equilibrium in the original model.

Necessary condition: if player 1 over-represents his valuation but just deviates locally, then  $\frac{w\sqrt{v_1}}{\sqrt{w}-\sqrt{v_1}}$  goes to infinity. Thus, if player 1 under-represents or over-represents locally, his problem at the beginning of the preliminary round is exactly the same as problem (8) above, so the necessary condition is already verified.

Sufficient condition: under the restrictions above, it is optimal to truthfully represent the valuation in problem (8). Thus, I have:

$$\begin{aligned} &v_1 F(v_1)^{N-1} \int_{\underline{v}}^{\bar{v}} \left( \sqrt{v_1} - \frac{v_B \sqrt{v_1}}{v_1 + v_B} \right)^2 dF(v_B)^N - b^P(v_1) \\ &> w F(v_1)^{N-1} \int_{\underline{v}}^{\bar{v}} \left( \sqrt{v_1} - \frac{v_B \sqrt{w}}{w + v_B} \right)^2 dF(v_B)^N - b^P(w), \quad \forall w \in [\underline{v}, \bar{v}], w \neq v_1 \end{aligned} \quad (10)$$

If player 1 under-represents his valuation, from (10), then the payoff is lower.

If player 1 over-represents his valuation, then

$$\begin{aligned} &v_1 F(v_1)^{N-1} \int_{\underline{v}}^{\bar{v}} \left( \sqrt{v_1} - \frac{v_B \sqrt{v_1}}{v_1 + v_B} \right)^2 dF(v_B)^N - b^P(v_1) \\ &> w F(v_1)^{N-1} \int_{\underline{v}}^{\bar{v}} \left( \sqrt{v_1} - \frac{v_B \sqrt{w}}{w + v_B} \right)^2 dF(v_B)^N - b^P(w) \\ &\geq w F(v_1)^{N-1} \int_{\underline{v}}^{\min\{\bar{v}, \frac{w\sqrt{v_1}}{\sqrt{w}-\sqrt{v_1}}\}} \left( \sqrt{v_1} - \frac{v_B \sqrt{w}}{w + v_B} \right)^2 dF(v_B)^N - b^P(w) \quad \forall w > v_1 \end{aligned} \quad (11)$$

Thus, from (11), it also decreases the payoff.

Hence, it is optimal to represent  $w = v_1$ .

**Q.E.D**

### Proof for Proposition 3

Denote  $T(v) = \int_{\underline{v}}^{\bar{v}} \frac{\zeta(v-\zeta)}{(v+\zeta)^3} dF(\zeta)^N$ , then  $S(v) = F(v)^{N-1} v T(v)$ .

It is not hard to see that  $T(\underline{v}) < 0$  and  $T(\bar{v}) > 0$ . Meanwhile,

$$\begin{aligned} T'(v) &= \int_{\underline{v}}^{\bar{v}} \frac{2\zeta(2\zeta-v)}{(v+\zeta)^4} dF(\zeta)^N \\ &\geq \int_{\underline{v}}^{\bar{v}} \frac{2\zeta(2v-\bar{v})}{(v+\zeta)^4} dF(\zeta)^N \\ &= \underline{v}(2-R) \int_{\underline{v}}^{\bar{v}} \frac{2\zeta}{(v+\zeta)^4} dF(\zeta)^N > 0 \end{aligned}$$

The last step follows the condition that  $R < 2$ .

Thus,  $T(v)$  is a strictly increasing function as well as  $T(\underline{v}) < 0$  and  $T(\bar{v}) > 0$ . So there exists a

threshold  $\underline{v} < v^* < \bar{v}$ , such that for  $\underline{v} \leq v < v^*$ ,  $T(v) < 0$ ; and for  $\bar{v} \geq v > v^*$ ,  $T(v) > 0$ .

For  $v = \underline{v}$ ,  $S(\underline{v}) = 0$ , so the marginal willingness to bid is the same.

For  $\underline{v} < v < v^*$ ,  $S(v) = F(v)^{N-1}vT(v) < 0$ , the marginal willingness to bid is lower in the presence of the signaling effect.

For  $v^* < v \leq \bar{v}$ ,  $S(v) = F(v)^{N-1}vT(v) > 0$ , the marginal willingness to bid is higher in the presence of the signaling effect.

**Q.E.D**

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