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The Extractive Firm's Cost Spillover Tax for the Extended Hotelling Model

John Hartwick
Queen's University

Andrei Bazhanov

Zhen Song

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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Andrei Bazhanov, John Hartwick and Zhen Song*

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Abstract

We consider a competitive extraction industry comprising many small firms, each with a slightly different quality of mineral holdings. With "rapidly" declining quality of holding per firm we observe rent declining over an interval. We then take up the familiar planning model and isolate the tax required to make decentralized extraction by many distinct, competitive firms replicate the planning solution.

- Keywords: exhaustible resources; resource rent; competitive extraction, corrective tax
- Journal classification: Q31, D41

1 Introduction

An issue of long-standing in oil-extraction economics is extending the Hotelling [1931] case of the competitive industry with identical small firms to one with small firms with distinct qualities of respective small holdings of stock. Linked to "the heterogeneous stock problem" is the question of whether observed rent across periods is declining or increasing. Livernois and Martin [2001] investigated the canonical (planning) model of industry extraction of a non-homogeneous stock (eg. Richard Gordon [1967], Cummings [1969], and Levhari and Leviatan [1977]) and argue that fairly standard assumptions allows one to rule out rent ever declining. Here we come at the question of industry extraction from the stand-point of the marginal extracting firm in the industry at any date. When

*Far Eastern National University, Vladivostok, Queen's University, Kingston, Ontario and Central University of Finance and Economics, Beijing. An earlier version was presented at a workshop at McGill University in January, 2008. We are indebted to participants for valuable comments.

we then move up to the competitive industry we observe that declining rent is a fairly natural outcome, given sufficient heterogeneity in endowments across extractive firms. Our result comes naturally because we work up from the level of the extractive firm rather than down from the level of the industry in a planning problem, as is standard in the literature (eg. Levhari and Leviatan [1977] and Livernois and Martin [2001]). A benchmark of our case as well as others is Hotelling's analysis.

We will distinguish two sorts of heterogeneous stock models. In the first, the value of the current stock is serving purely as an address of where current extraction, $Q(t)$ is occuring in the problem. For this case one firm's extraction has no spillover into the costs of the firms that follow. This sort of model we report on first. There is heterogeneity of stock holding across firms but no cost externality. This type of model seems new to the literature. Our illustration is that of a deep pipe of oil with each ton owned by a distinct firm, a ton being small relative to the total stock. The oil can be intrinsically homogeneous but since each ton is more expensive to extract than that preceding, we characterize the setting as distinct qualities across firms with quality declining as the deposit is extracted from (top down over time). We demonstrate that this sort of model allows for non-trivial intervals of declining resource rent. Roughly speaking the "more heterogeneity" in oil stock holdings across firms, the more prevalent is the interval of declining rent. In the second sort of model, one firm's current extraction shifts the cost function for each subsequent firm doing extraction. For example, the current firm's extraction can draw off pressure from an oil field and drive up the extraction cost of each firm following. The pressure draw-off can be thought of as varying with the quantity path of the oil drawdown from the deposit. In this case the "prices" in a firm's extraction cost function can be viewed as not parametric to the firm and an externality is operative. This sort of heterogeneous stock model is standard in the literature and the problem of the externality has been explicitly recognized (Richard Gordon[1] [1967], Cummings

[1] Richard L. Gordon [1967; p. 282] is explicit that the solution (for heterogeneous extractive frims) with "efficient socialist managers" differs from that"of individual firms in a competitive industry". He fails to address the question of a corrective tax. Cummings reflects on Gordon's planning solution for the problem with heterogeneous firms and also fails to address the matter of a corrective tax. The idea of a market failure does not appear in the analyses of Levhari and Leviatan and of Livernois and Martin.

[1969]) though we know of no presentation of a tax which internalizes the cost-spillover externality across firms. We report on the required tax here. Much of our analysis is carried out in a framework of discrete time and we feel that this approach makes the analysis clearer than it would be if we worked in continuous time. We are nonetheless about to specify the form of the required tax for the familiar continuous time model. It is also true that by working with our first sort of model, we are able to make the analysis of the "traditional" model and its corrective tax clearer. Livernois and Martin [2001] have proved that rent cannot decline over time in the "traditional" model with standard assumptions, under central planning. Since planning models are rarely implemented in the real world, we doubt that predictions from such models about rent paths have much utility. Part of our analysis deals with models that "permit" declining rent paths for the real world.

When our result contradicts the central result of Livernois and Martin we are not indicating that the details of their analysis are faulty, rather we suggest that the concept of industry equilibrium which they work with, one "embedded in" a planning framework, may not be good at capturing the problem of extraction with many competitive firms, each with a distinct small holding of the total stock. We wish to make the point here that declining exhaustible resource rent may be a fairly natural property of the data on resource rents. We return to the matter of the appropriate model below.

2 The Analysis

We focus first on the condition for zero profit arbitrage for the marginal firm in period t. Our set up involves each firm with a distinct extraction cost and each firm owning a very small amount of oil, say 1 ton for concreteness. In our industry equilibrium there will be Q(t) firms extracting at date t and the marginal firm, will have extraction costs for its ton as $C(Q(t), S(t))$. Each other firm extracting at date t will have extraction costs lower than $C(Q(t), S(t))$. Hence the marginal firm will earn a "Hotelling" rent on its unit extracted whereas each intramarginal firm which is active at the same instant will earn the same

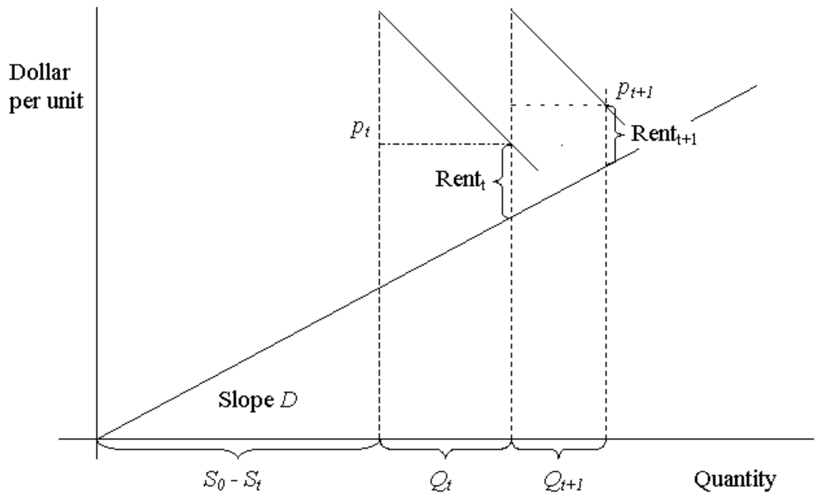

Figure 1: $Q(t)$ in period t and $Q(t+1)$ in period t+1, with the marginal firm in period t being the intra marginal firm in period t+1. For the marginal firm to be indifferent between extracting between periods, its profit on its ton must differ between periods by $r\%$. Hence $p(t+1) - C(Q(t), S(t)) = (1+r)[p(t) - C(Q(t), S(t))]$.

"Hotelling" rent plus some "quality" (Ricardian) rent. In Figure 1 we illustrate extraction of $Q(t)$ firms at date t and $Q(t+1)$ firms t+1 in a discrete time "approximation".

In Figure 1, we have the arbitrage condition for the marginal firm in period t, in

$$p(t+1) - C(Q(t), S(t)) = (1+r)[p(t) - C(Q(t), S(t))].$$

In continuous time this zero profit arbitrage condition for a firm is taken as

$$\dot{p}(t) = [p(t) - C(Q(t), S(t))]r. \tag{1}$$

r is the constant discount (interest) rate.

We turn to an example in order to illustrate industry equilibrium and rent declining over time. We take $C(Q(t), S(t))$ as $[S_0 - S(t) + Q(t)]D$ for D a positive constant and S_0 the initial stock comprising S_0 single-ton holdings by S_0 distinct firms. Think of the S_0 tons in a long geological pipe vertical in the ground. It is cheaper to extract any ton closer to the surface. $Q(t)$ is then the number of firms extracting at date t. (This implies that it is appropriate to view our $[S_0 - S(t) + Q(t)]D$ schedule as a step function, with each runner one ton wide.[2]) We assume that the market (industry) demand schedule is linear as in $p(t) = A - BQ(t)$ for A and B positive constants. For end date T, we take $Q(T) = 0$ and terminal rent as $A - S_0 D > 0$.[3] Then $Q(T-1)$ gets defined from $p(T-1) - [S_0 - S(T-1) + Q(T-1)]D = \left[\frac{1}{1+r}\right][A - S_0 D]$. (For the special case of $D = 0$ we have Hotelling's original competitive industry with his linear demand schedule.) Then industry equilibrium involves solving

$$\begin{aligned} -B\dot{Q}(t) &= [A - BQ(t) - [S_0 - S(t) + Q(t)]D]r \\ \text{and } \dot{S}(t) &= -Q(t). \end{aligned}$$

Note that the first equation above indicates that $-\dot{Q}(t)$ is proportional to current rent, the factor of proportionality being r/B. Thus if rent is declining, $-\dot{Q}(t)$ will be declining. In textbook renderings of Hotelling's competitive industry model, $-\dot{Q}(t)$ is usually indicated to be increasing with time and we observe this behavior for the case of D near zero. The "geology" of this example was sketched in the introduction. One thinks of S_0 firms each owning one unit of homogeneous stock, with each unit stacked on others as with a straight pipe moving down toward the center of the earth. The pipe would have depth S_0. "Early" firms face lower extraction costs because their holding is less deep than "later" firms. The cost of extraction per firm increases in a linear fashion as "the process" moves deeper as time passes.

The above pair of equations becomes the second order linear equation in $S(t)$:

$$\frac{d\dot{S}}{dt} + a_1\dot{S} + a_2 S = R_0. \tag{2}$$

[2] Each firm sees itself as having a constant unit cost extraction schedule and each firm's schedule is distinct.

[3] A different end-point condition has rent equal to zero at the end and some high-cost stock left unextracted. We work with the case of all stock extracted and terminal rent positive. We address the question of fitting an initial stock, S_0 into a finite number of time slots by solving backwards in time and thus "choosing" our S_0 to be exact for our given number of time slots. Obviously the question of fitting a given S_0 to a given number of time slots exactly is peripheral to our analysis here. See the interesting analysis of Lozda [1993] on the matter of fitting a given "arbitrary" S_0 into a finite and "correct" number of time slots.

for $a_1 = -\left[\frac{B+D}{B}\right] r$, $a_2 = -\left[\frac{Dr}{B}\right]$ and $R_0 = \left[\frac{A-S_0D}{B}\right] r$. $A - S_0D$ is rent corresponding to $Q(T) = 0$ and $S(T) = 0$ and must be non-negative. We turn to solve equation (2) in S. Note that the limits of the model require that the values of parameters A, S_0, D and r are such that $R_0 > 0$. Otherwise, for $R_0 = 0$ we have trivial solution $S(t) \equiv 0$ and for $R_0 < 0$ we have negative $S(t)$ for all $t \in (0, T)$.

In the simplest case, the boundary conditions will be $S(T) = 0$ and $\dot{S}(T) = -Q(T) = 0$. Hence the solution involves obtaining the $Q(0)$ which results in the boundary condition being satisfied at end date T.[4] In our case the solution of (2) is

$$S(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + F$$

because the determinant of the characteristic equation $\Delta = a_1^2 - 4a_2$ for the corresponding homogeneous equation is always positive in our problem. We have $\lambda_1 = (-a_1 + \sqrt{\Delta})/2$ and $\lambda_2 = (-a_1 - \sqrt{\Delta})/2$ - the roots of the characteristic equation and $F = R_0/a_2$ is a particular solution of the nonhomogeneous equation (2). Using terminal condition $\dot{S}(T) = 0$ we have $C_1 = -C_2\lambda_2 e^{T(\lambda_2-\lambda_1)}/\lambda_1$. Substituting it into (2) and using the initial condition $S(0) = S_0 = C_2\left[1 - \lambda_2 e^{T(\lambda_2-\lambda_1)}/\lambda_1\right] + R_0/a_2$ we obtain

$$C_2 = \frac{S_0 - R_0/a_2}{1 - \frac{\lambda_2}{\lambda_1} e^{T(\lambda_2-\lambda_1)}} \text{ and } C_1 = -\frac{S_0 - R_0/a_2}{\frac{\lambda_1}{\lambda_2} e^{T(\lambda_1-\lambda_2)} - 1}.$$

Then we can use the second terminal condition $S(T) = 0$ in order to define T :

$$C_2(T)e^{\lambda_2 T} + C_1(T)e^{\lambda_1 T} + R_0/a_2 = 0$$

or

$$\frac{S_0 - R_0/a_2}{1 - \frac{\lambda_2}{\lambda_1} e^{T(\lambda_2-\lambda_1)}} e^{\lambda_2 T} - \frac{S_0 - R_0/a_2}{\frac{\lambda_1}{\lambda_2} e^{T(\lambda_1-\lambda_2)} - 1} e^{\lambda_1 T} + \frac{R_0}{a_2} = 0$$

which after transformations can be written as follows

$$e^{-\lambda_2 T} = R_1 + \frac{\lambda_2}{\lambda_1} e^{-\lambda_1 T} \tag{3}$$

where $R_1 = (1 - a_2S_0/R_0)(1 - \lambda_2/\lambda_1)$. Nonlinear equation (3) in T has a unique positive solution (Fig. 2) because $\lambda_2 < 0, a_2 < 0$ which follows $R_1 > 1$.

[4] A more complicated case will have price rise relatively rapidly and some high cost firms will choose not to extract because their profit would be negative.

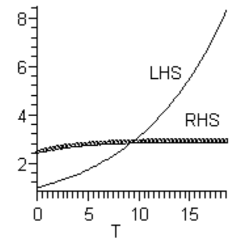

Figure 2: Solution of equation in T.

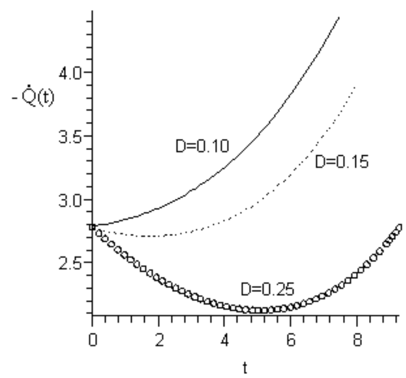

Figure 3: "Acceleration" of extraction, $-\dot{Q}(t)$ for $D = 0.1$ (solid line), $D = 0.15$ (dotted line), and $D = 0.25$ (in circles).

This implies that we have in the left hand side an exponentially increasing function starting from unity and in the right hand side an increasing function starting from the value which is greater than unity but less than R_1 and asymptotically approaching R_1.

We report on numerical example with $r = 0.1, A = 50$, and $B = 0.9$ that growth in D follows the inverted hump-shaped accelerations $-\dot{Q}(t)$ (Fig. 3)[5]. This indeed implies the decreasing pattern of rent in a neighborhood of initial point $t = 0$ for $D = 0.25$ (Fig. 4).

The corresponding behavior of extraction $Q(t)$ and price $p(t)$ are depicted

[5] For the values of $D = 0.1$, $D = 0.15$, and $D = 0.25$ equation (3) implies $T_{0.1} = 7.45$, $T_{0.15} = 7.94$, and $T_{0.25} = 9.28$.

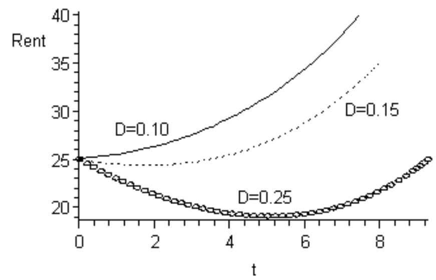

Figure 4: Rent function for $D = 0.1$ (solid line), $D = 0.15$ (dotted line), and $D = 0.25$ (in circles).

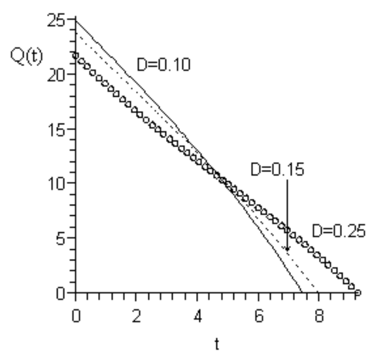

Figure 5: Extractions.

on figures 5 and 6.

Observe that quantity and price paths in Figures 5 and 6 each have a point of inflection for $D = 0.15$ and $D = 0.25$.

3 Planning Solution versus Market Solution

The textbook planning solution (Levhari and Leviatan [1977] and Livernois and Martin [2001]) works with an extraction cost function defined on current industry output, $Q(t)$ in $C(Q(t), S(t))$. $S(t)$ is the current stock remaining. It $Q(t)$ is viewed as say $Q(t)$ distinct firms extracting at date t , then it is standard, given this approach to quality change across firms to view each firm's extraction

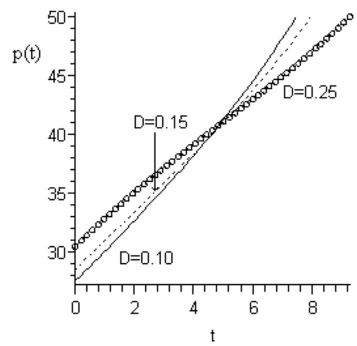

Figure 6: Price.

as affecting the cost of extraction of every firm following the one in question. We assume that $\frac{\partial C(t)}{\partial Q(t)} > 0$ and $C(0, S(t)) = 0$; and $\frac{\partial C(t)}{\partial S(t)} < 0$. The physical account involves $Q(t) = S(t) - S(t+1)$. The planner has of course perfect foresight and is able to vary current industry output in order to "smooth" extraction costs over periods by varying industry output across periods with a view to maximizing the present value of surpluses. We will represent current gross consumer surplus as $B(Q(t))$, with $p(t) = \frac{dB(Q(t))}{dQ(t)}$. For our discrete time formulation, the planner is assumed to maximize

$$
\begin{aligned}
W \quad = \quad & [B(Q(0)) - C(Q(0), S(0))] + \left[\frac{1}{1+r}\right] [B(Q(1)) - C(Q(1), S(1))] \\
& + \left[\frac{1}{1+r}\right]^2 [B(Q(2)) - C(Q(2), S(2))] \\
& + \left[\frac{1}{1+r}\right]^3 [B(Q(3)) - C(Q(3), S(3))] \\
& + ... \\
& + \left[\frac{1}{1+r}\right]^T [B(Q(T)) - C(Q(T), S(T))]
\end{aligned}
$$

where the $Q(t)'s$ are the control variables and $S(0)$ is finite. The necessary condition (Euler equation) for this problem is

$$
\begin{aligned}
p(t) - C_Q(q(t), S(t)) \quad = \quad & \left[\frac{1}{1+r}\right] [p(t+1) - C_Q(Q(t+1), S(t+1))] \\
& - \left[\frac{1}{1+r}\right] C_S(Q(t+1), S(t+1))
\end{aligned}
$$

where $C_S(Q(t+1), S(t+1))$ is usually referred to as "the stock size" effect.[6] The other terms represent an $r\%$ rule in rent across periods. Hence the necessary condition is an "amended" Hotelling Rule.[7]

We can re-write this condition by adding and subtracting $\left[\frac{1}{1+r}\right] C_Q(\varepsilon, S(t+1))$ (for ε a small positive quantity representing the marginal firm's quantity at date t) to the right-hand side to get

$$p(t) - C_Q(Q(t), S(t)) = \left[\frac{1}{1+r}\right] [p(t+1) - C_Q(\varepsilon, S(t+1)) + \tau(t+1)]$$

where $\tau(t) = [C_Q(\varepsilon, S(t+1)) - C_Q(Q(t+1), S(t+1)) - C_S(Q(t+1), S(t+1))]$ is the tax required for decentralizing the socially optimal solution. "On its own" the marginal firm at date t would extract to equalize $p(t) - C_Q(Q(t), S(t))$ and $\left[\frac{1}{1+r}\right] [p(t+1) - C_Q(\varepsilon, S(t+1))]$. Recall the argument in the previous section. Hence the need for a tax. Since $C_S(.,.)$ is assumed to be negative and $C_Q(Q, S)$ is assumed to be increasing in Q, this tax could be of either sign and possibly be close to zero.

In the previous section (our model with no cost spillovers between firms) we had essentially $C_Q(\varepsilon, S(t+1)) = C_Q(Q(t), S(t))$ and this seems to be an empirically plausible assumption to make. This is indicating that the marginal firm at date t has approximately the same extraction cost for its ton as the intra-marginal firm at date $t+1$. The marginal firm at t and the intramarginal firm at date $t+1$ are assumed to be very close neighbors in the array of firms in the stock. $C_Q(\varepsilon, S(t+1)) = C_Q(Q(t), S(t))$ would obtain, given an assumption on smoothness in cost increases across "close" holdings by distinct firms in the oil field. Given smoothness across costs for neighboring firms, the appropriate tax becomes

$$[C_Q(Q(t), S(t)) - C_Q(Q(t+1), S(t+1)) - C_S(Q(t+1), S(t+1))]. \qquad (4)$$

[6] In a continuous time analysis, the corresponding Euler equation works out as $\frac{d[p(t)-C_Q(t)}{dt} = [p(t) - C_Q(t)]r + C_S(t)$.

[7] For the end point we assume that S_0 is exhausted and terminal rent is positive. We also assume that S_0 fits exactly into the "correct" number of time periods. End point conditions for such discrete time formulations are tricky because one is fitting a finite stock into a finite number of periods, with the initial stock of arbitrary size. Informally we have a cookie-cutter problem in this "fittting problem". See Lozada [1993] for the analysis of a related problem.

The continuous time version of the problem (Levhari and Leviatan [1977]) is

$$\max_{\{Q(t)\}} \int_0^T [B(Q(t)) - C(Q(t), S(t))]e^{-rt}dt$$

$$\text{subject to } \dot{S}(t) = -Q(t) \text{ and } \int_0^T Q(t)dt \leqq S_0.$$

The necessary condition (Euler equation) for this problem is

$$\dot{p}(t) = [p(t) - C_Q(t)] + \dot{C}_Q(t) + C_S(t).$$

Obviously $\dot{C}_Q(t) + C_S(t)$ is an approximation to $-[C_Q(Q(t), S(t)) - C_Q(Q(t+1), S(t+1)) - C_S(Q(t+1), S(t+1))]$ in (4) and the expression in (4) is a candidate for the corrective tax $\tau(t)$. Hence we can express the Euler equation for the continuous time problem as

$$\dot{p}(t) = [p(t) - C_Q(t)]r - \tau(t) \tag{5}$$

for $\tau(t) = -[\dot{C}_Q(t) + C_S(t)]$. Thus one can envisage an appropriate corrective tax, even in the standard continuous time model, though our analysis has been made easier by approaching the issue of sustaining the socially optimal solution first off in a discrete time setting. If one compares equation (5) with equation (1), one sees how the model in the preceding section is a version of our model here, only "free" of a corrective tax. We constructed our model in the previous section to have no cost spillover between firms and hence no need for a tax.

Casual empiricism suggests that, in general, extraction around the world is not guided by a tax structured along the lines above. A reasonalbe inference is that extraction follows an essentially second best path, a path with spillovers in costs across firms uncorrected. Presumably the welfare cost of this "lapse" in first best pricing is not high.

4 Concluding Remark

We have constructed an example of extraction by many firms with a small holding of oil, each holding distinct, which has Hotelling [1931] as a special case. The solved example fails to exhibit steadily increasing rent over time, the outcome that Livernois and Martin [2001] established as the inevitable outcome in their model, a model in the tradition of Richard Gordon [1967], Cummings [1969] and Levhari and Leviatan [1977] (a planning solution to the industry extraction problem). Our aim is to establish that in a plausible model of competitive extraction, heterogeneity of stock can lead in an unforced way to periods of declining rent. Our model was carefully specified to have no cost spillovers across firms. We then turned to the "traditional" model with essential cost spillovers across firms and its planning solution. We proceeded to obtain the corrective tax which is necessary at each date in order for a decentralized solution to mimic the optimal planning solution. Our final point was that corrective taxes are not in effect in the real world for cases analogous to those under study here and hence that the real world is a "second best morass", one that probably exhibits dead weight losses sufficiently small to not merit the complicated interventions that are required to sustain first best extraction paths.

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