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The Role of Large Players in a Dynamic Currency Attack Game

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Abstract

We establish a dynamic currency attack model in the presence of a large player (LP) based on Abreu and Brunnermeier (2003), which differs from most existing one-period static currency attack models. In an attack on a fixed exchange rate regime with a gradually overvaluing currency, both the inability of speculators to synchronize their attack and their incentive to time the collapse of the regime lead to the persistent overvaluation of the currency. We find that the presence of an LP, who is defined as a speculator with more wealth and superior information, can accelerate or delay the collapse of the regime, depending on his incentives to preempt other speculators or to "ride the overvaluation". When an LP's incentive to preempt other speculators is dominant, the presence of an LP will accelerate the collapse of the regime. However, when an LP's incentive to "ride the overvaluation" is dominant, the presence of an LP will delay the collapse of the regime. The latter case provides valuable insights into the role that LP's play in currency attacks: it differs from the usual perception that the presence of LPs will facilitate arbitrage in an asset market and alleviate asset mispricing due to their capability and willingness to arbitrage.

Keywords: Large Player, Currency Attack

JEL Classification: D80, F31

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1 Introduction

We often observe that large players such as hedge funds play an active role in currency attacks against fixed exchange rate regimes. They launch currency attacks by employing a large amount of wealth to build large short positions. Afterward they try to influence market sentiment by publicly announcing their short positions and beliefs that devaluation is inevitable. This causes herding among small traders, and/or deters contrarians from taking opposite positions. It seems that the presence of large players facilitates coordination among speculators and increases financial instability in the attacked currencies, specially in small economy currencies. This is sometimes called the "big elephants in small ponds" effect.

We establish a formal model to study the role that large players play in a currency attack, based on the model developed by Abreu and Brunnermeier (2003). In their model, rational arbitrageurs in an asset market become aware of an asset bubble sequentially. Due to the lack of common knowledge about the bubble, and need for coordination to burst the bubble, the bubble will be persistent and its bursting time depends on the incentives of the arbitrageurs to " ride the bubble," as opposed to incentives to preempt other arbitrageurs in selling the asset. Similar to their model, we assume that a currency begins to be overvalued in a fixed exchange rate regime after a certain time. Speculators have dispersed opinions in the sense that they only become aware of the overvaluation sequentially. In addition, we assume that the fixed exchange rate regime will collapse only when attacking pressure reaches a threshold level. This assumption captures the main feature of currency attacks: there is a necessity for coordination among speculators to break a currency peg. This coordination feature is emphasized in Obstfeld (1996) and other currency attack models (see especially Morris and Shin (1998)). In our setup, the speculators try to choose the optimal time to launch their attack, driven by two competing incentives: first, the incentive to "ride the overvaluation;" and second, the incentive to preempt other speculators. The speculators' incentive to "ride the overvaluation" stems from two sources in our model: first, they can reap higher benefits from the devaluation if the overvaluation lasts longer. Second, if they time their attack more precisely, they will save on attacking costs. The speculator attempts to preempt other speculators, because only the speculator attacking early will gain from the collapse of the regime. The late speculators will gain nothing.

Abreu and Brunnermeier (2003) consider a symmetric game with a continuum of atomistic small arbitrageurs. We are more interested in a richer market structure where both a large player and a continuum of small players are present. More specifically, we are interested in studying how the presence of a large player will change equilibrium outcomes. In our model, a large player is defined by two characteristics: first, he has more precise information about the fundamental value of a currency. Here we assume the extreme case where a large player has perfect information about the time when the overvaluation begins. Second, a large player can employ substantially larger amounts of wealth to launch a currency attack. The wealth that a large player employs can come from his own capital, or more importantly, from his accessibility to credit due to his reputation. This is how highly leveraged financial institutions finance their speculation.

Our model differs from most currency attack literature in several aspects. First, it is one of the few papers studying currency attacks in a dynamic setup. Most existing literature uses static models, which miss the complicated market dynamics in currency attacks. Second, while most existing currency attack literature simply assumes exogenously the existence of the overvaluation under a fixed exchange rate regime, we model endogenously the origin of currency overvaluation in the presence of rational arbitrageurs. A key contribution of Abreu and Brunnermeier (2003) lies in that they offer a general explanation of how asset mispricing arises. Even in the presence of rational arbitrageurs, who are capable of correcting the mispricing, they choose not to do it due to their incentive to "ride the bubble." This mechanism can be comfortably applied to explaining how currency overvaluation arises in a fixed exchange rate regime.

Our approach is consistent with the microstructure method for modeling exchange rates. We believe that foreign exchange market participants hold highly dispersed opinions about exchange rates. As argued by Lyons (2001), even if all market participants have the same information about exchange rates, the ways in which they interpret or model the implications of that information can be different. Thus, they can come to different conclusions about exchange rates based on the same information. So it is well justified to assume that speculators do not have perfect information about the time when the overvaluation arises, and only become gradually aware of the overvaluation. In such an opinion-dispersed market where coordination is required for a successful currency attack, the incentive for "riding the overvaluation" naturally arises and leads to the persistent overvaluation of the currency. This explanation is consistent with empirical observations that currency attacks often lead to substantial and sudden devaluations, causing extreme volatility in an economy.

Due to the features of our model, our study of large players in currency attacks focuses on a different aspect compared to the standard, more static models. Most existing literature on large players in currency attacks focuses on the possibility of the collapse of a fixed exchange rate regime, and on whether the presence of large players will increase this possibility or not. In our model, currency devaluation is inevitable, and the issue that we focus on is *when* it will happen. Thus our study focuses on whether the presence of large players will *accelerate* or *delay* a currency attack. Here we do not give a formal welfare analysis to examine whether the presence of a large player is beneficial or harmful to an economy. However, in general, we believe that a currency overvaluation is harmful to an economy, and early correction is always better than a late one if the correction is inevitable. In this sense, a late collapse of the regime will do more harm to an economy than an early one.

Using our model, we find some interesting results. First, we find that the presence of a large player will not necessarily accelerate the collapse of an overvalued fixed exchange rate regime. This result is important because large players are usually believed to facilitate arbitrage in an asset market and reduce asset mispricing. In our model, the presence of a large player will *accelerate* or *delay* the collapse of a fixed exchange rate regime, depending on whether his incentive to "ride the overvaluation" is dominated, or not, by his incentive to preempt the mass of small speculators. If his incentive to preempt is dominant, his presence will accelerate the collapse of the regime. He can do so not only because his wealth facilitates the attack, but also because his presence makes other small speculators attack earlier. Conversely, if a large player's incentive to preempt other speculators is dominated by the incentive to

“ ride the overvaluation,” his presence will delay the collapse of the regime. He can do so not only because he will wait longer, but also because his presence makes other speculators wait longer too. The large player’s incentive to “ride the overvaluation” makes the existence of a large player a mechanism for delaying any asset mispricing. Instead, he will use his market power to make greater profits from larger, later asset mispricing.

The rest of the paper consists of six sections. Section 2 provides a literature survey. Section 3 discusses the basic model and characterization of a dynamic currency attack. The model is a variation of that established by Abreu and Brunnermeier (2003): the model has a continuum of small arbitrageurs trading in a currency with a fixed exchange rate, that is open to a currency attack. We provide a characterization of the equilibrium and comparative statics. Section 4 introduces a large player, proves that there is a unique equilibrium and characterizes that equilibrium. Section 5 conducts comparative statics for the model. Section 6 observes that our results can be applied to the original Abreu and Brunnermeier (2003) set-up with a stock market. Section 7 concludes with observations on further possible extensions.

2 Literature Survey

Abreu and Brunnermeier (2003) construct a dynamic coordination game to explain the existence of asset bubbles, even in the presence of rational arbitrageurs who are capable of bursting the bubble. We have already discussed their model in detail. (Our paper is an application of their model to currency attacks.) In their model, only a continuum of atomistic speculators exists. Since we focus on the study of the role that a large player plays in a currency attack, our model exhibits a richer market structure where both a large player and a continuum of atomistic speculators co-exist.

Both Rochon (2006) and Gara Minguez-Afonso (2007) apply Abreu and Brunnermeier (2003) to currency attacks and try to explain the devaluation that we observe when a fixed exchange rate regime collapses. The most important difference between our model and theirs is that our model focuses on the role of large players in a currency attack with imperfect common knowledge, while they study currency attacks only in a model without large players. In addition, even in our basic model without large players, the way in which we model a currency attack is also slightly different from theirs. We model the payoff structure of speculators who try to gain from the devaluation, while they model the payoff structure of the attackers who try to avoid a capital loss associated with devaluation.

Morris and Shin (1998) study currency attacks in a one-period global game setup. They demonstrate that, although a self-fulfilling currency attack game has multiple equilibria when economic fundamentals are common knowledge, it has a unique equilibrium when speculators can only observe the fundamentals with small noise. Successfully overcoming the problem of indeterminacy of multiple equilibria models, their model allows the analysis of policy implications.

Corsetti, Dasgupta, Morris and Shin (2004) extend the model established by Morris and Shin (1998) to one with a large player. They analyze two cases where the large player has, and has not, a signalling function. They find that in both cases the presence of a large player does increase the possibility of the collapse of a fixed exchange rate regime, and make small speculators more aggressive.

Correstti, Pesenti, and Roubini (2001) give a comprehensive survey on the role that large players play in currency attacks. In the theoretical section of their survey, they apply a traditional coordination game with perfect information, and then a global game established by Corsetti, Dasgupta, Morris and Shin (2004) to the study of the role of a large player in a currency market. In the empirical section, they combine both econometric analysis and case studies to explore examples of currency attacks. Their conclusion is that both theoretical and empirical studies reveal that large players do have a significant role in currency attacks, and more academic research is required to address a number of issues, including the dynamics of currency attacks or crises.

Bannier (2005) modifies the model established by Corsetti, Dasgupta, Morris and Shin (2004) by changing the assumption about a central bank's strategy. Due to that modification, both the large player and small speculators' strategies are symmetric and analytical results are available. She finds that this modification changes the results given by Corsetti, Dasgupta, Morris and Shin (2004). Now a large player can increase the possibility of a regime collapse only when market sentiment is pessimistic.

However, the presence of a large player will decrease the possibility of a regime collapse when the market sentiment is optimistic.

3 The Benchmark Model without a Large Player

3.1 Environment

This model is a simple modification of the Abreu and Brunnermeier (2003) model. We capture the essence of their idea that the difficulty in coordination among arbitrageurs, together with their incentive to time the market, can cause asset mispricing. We modify the model to apply it to foreign exchange markets.

Assume that there is a country with a fixed exchange rate regime where a central bank commits to maintaining the exchange rate at a fixed level until it exhausts all of its foreign reserves, whose level is denoted by $k > 0$.

From time $t_0 > \eta$, the exchange rate becomes overvalued relative to its fundamental value, at a rate of g. Denote the initial exchange rate as E_0. The fundamental exchange rate at t is E_0 when $t < t_0$ and $E_0(1 + g(t - t_0))$ when $t \geq t_0$. Here the exchange rate is denominated in the domestic currency, say wons. So E_0 means that 1 dollar can exchange for E_0 wons.

Without any currency attacks, the fixed exchange rate regime will collapse at some exogenously given time $t_0 + \tau'$. This assumption captures the idea that any asset mispricing is not sustainable in the long run. We follow Abreu and Brunnermeier (2003) in making this simplified assumption to avoid ever greater currency overvaluations. Figure 1 shows how the fundamental exchange rate changes with time.

There is a continuum of atomistic speculators of mass 1. Each speculator is financially constrained and can only access the credit whose worth is normalized to 1 dollar. Each speculator has to choose from two strategies: attacking or refraining. When $t < t_0 + \tau'$, the exchange rate will devalue to the fundamental value if and only if attacking pressure exceeds k. This assumption follows that of Obstfeld (1996) and Morris and Shin (1998) and captures the idea of market liquidity.

We specify the payoff structure of speculators as follows: if they choose refraining,

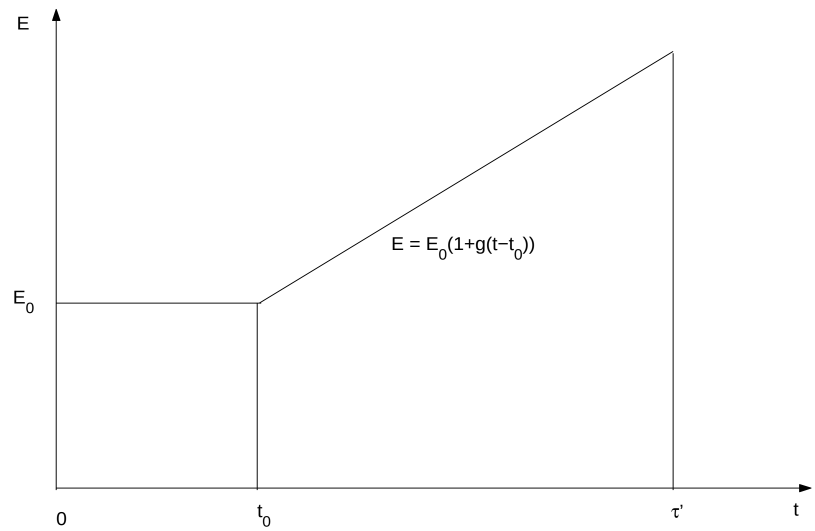

Figure 1: How the fundamental exchange rate E changes with time t

which means that they will do nothing, they will gain zero. If they choose to attack, they will borrow wons from the banks of the attacked country, then exchange them into dollars from the central bank. The costs of attacking consist of two parts. One part is the fixed transaction costs associated with the currency exchanges, which is denoted by c^F. We assume that the fixed transaction costs are not so high that they prevent the speculators from ever attacking, despite the awareness of the overvaluation. The other part is the interest differential between wons and dollars, since we assume that the interest rate of wons is higher than that of dollars. Let c denote the interest differential. Thus, if a speculator keeps attacking during a time interval $\triangle t$, he will incur the cost of $c.\triangle t$. The payoffs of speculators from attacking is as follows. If the regime collapses at instant t, the payoffs of a speculator attacking at instant t with the wealth of 1 dollar will depend on how many other speculators are attacking. If the attacking mass is less than or equal to k, his payoffs are $E_0.g(t-t_0)$. If the attacking mass is greater than k, only the first randomly chosen mass k of attacking speculators will gain the payoffs of $E_0.g(t-t_0)$. So given the attacking pressure $\alpha > k$, the expected payoffs of a speculator are given by $\frac{k}{\alpha}E_0.g(t-t_0)$. For simplicity of the analysis, we assume that no partial attacking is allowed.

The speculators only have imperfect information about t_0, the time at which the overvaluation begins. More specifically, all the speculators have a prior belief about t_0, which is denoted by $\Phi(t_0)$. We assume that the speculators have an improper uniform belief about t_0 over $[0, \infty)$.

From t_0, a new cohort of small speculators with mass $\frac{1}{\eta}$ becomes aware of the overvaluation in each instant from t_0 until $t_0 + \eta$.

Conditional on t_i, speculator t_i's belief about t_0 is given by the CDF

$$\Phi(t_0|t_i) = \frac{t - t_i + \eta}{\eta}, \tag{1}$$

where $t \in [t_i - \eta, t_i]$.

Given such a setup, we try to find the equilibrium strategy of a rational speculator t_i.

Let $\sigma(t, t_i)$ denote the strategy of speculator t_i and the function $\sigma : [0, \infty) \times [0, \infty) \mapsto \{0, 1\}$ a strategy profile. Speculator t_i's strategy is given by $\sigma(., t_i) : [0, t_i + \tau'] \mapsto \{0, 1\}$, where 0 means refraining and 1 means attacking. The aggregate attacking pressure of all the speculators at time $t \geq t_0$ is given by

$$s(t, t_0) = \int_{t_0}^{min\{t, t_0+\eta\}} \sigma(t, t_i) dt_i. \tag{2}$$

Let

$$T^*(t_0) = inf\{t|s(t, t_0) \geq k \text{ or } t = t_0 + \tau'\} \tag{3}$$

denote the collapse time of the fixed exchange rate regime for a given realization of t_0. Recall that $\Phi(.|t_i)$ denotes speculator i's belief about t_0 given that $t_0 \in [t_i - \eta, t_i]$. Hence, his belief about the collapse time is given by

$$\Pi(t|t_i) = \int_{T^*(t_0)<t} d\Phi(t_0|t_i).$$

The time t_i expected payoffs of speculator t_i, who remains refraining until he begins to attack at time t and keeps attacking afterward until the regime collapses, are given by

$$\int_t^{t_i+\tau'} E_0 g(s - T^{*-1}(s)) - c(s - t) d\Pi(s|t_i) - c^F,$$

provided that the attacking pressure at t does not strictly exceed k and that $T^*(.)$ is strictly increasing. Later we will show that in equilibrium all the conditions will hold.

If we normalize the initial exchange rate to 1, we get:

$$\int_t^{t_i+\tau'} g(s - T^{*-1}(s)) - c(s-t)d\Pi(s|t_i) - c^F. \tag{4}$$

3.2 Equilibrium Characterization

We confine our attention to symmetric trigger strategies. We can prove that there is a unique symmetric trigger strategy equilibrium. In this equilibrium, each speculator t_i will attack at the instant $t_i + \tau^*$ and keep attacking until the regime collapses. Depending on parameter values of η, k, g and c, the regime can collapse exogenously or endogenously. Here we will focus on the endogenous collapse case.

Rochon (2006) proves in a similar setup that this symmetric trigger strategy equilibrium is a strongly rational expectation equilibrium in the set of strategies with the only restriction being that speculators act after being informed.

Proposition 1. *Given $\tau' > \frac{c}{g}k\eta$ and $c \geq g$, there is a unique symmetric trigger strategy equilibrium where the regime collapses endogenously. In this equilibrium, each speculator t_i begins to attack at the instant $t_i + \tau^*$ and keeps attacking until the regime collapses, where $\tau^* = \frac{c-g}{g}k\eta$. In equilibrium the regime collapses exactly at the instant $t_0 + k\eta + \tau^*$.*

Given $\tau' > k\eta$ and $c < g$, there is a unique symmetric trigger strategy equilibrium where the regime collapses endogenously. In this equilibrium, each speculator t_i begins to attack at the instant t_i and keeps attacking until the regime collapses. In equilibrium the regime collapses exactly at the instant $t_0 + k\eta$.

Proof:

Let τ^* define a symmetric trigger equilibrium. That is, all the speculators begin to attack at $t_i + \tau^*$. Given such a strategy, the regime will collapse when speculator $t_0 + k\eta$ attacks, and the collapsing time will be $t_0 + k\eta + \tau^*$.

Now consider the optimal strategy of speculator t_i given that all the other speculators take the strategy τ^*. Thus the regime will collapse at $t_0+\zeta$, where $\zeta = k\eta+\tau^*$. Speculator t_i believes that $t_0 \in [t_i-\eta, t_i]$, the CDF of his posterior belief about t_0 is given by

$$\Phi(t|t_i) = \frac{t-t_i+\eta}{\eta}. \tag{5}$$

Since the collapsing time is $t_0+\zeta$, he believes that $t_0+\zeta \in [t_i-\eta+\zeta, t_i+\zeta]$. The CDF of his posterior belief about the collapsing date $t_0+\zeta$ at time $t_i+\tau$ is given by

$$\Pi(t_i+\tau|t_i) = \frac{t_i+\tau-(t_i-\eta+\zeta)}{\eta} = \frac{\tau+\eta-\zeta}{\eta}. \tag{6}$$

Speculator t_i's expected payoff from attacking at t and keeping attacking until the regime collapses is given by:

$$\int_t^{t_i+\zeta} (g(s-T^{*-1}(s)) - c(s-t))d\Pi(s|t_i) - c^F. \tag{7}$$

The first order condition gives the optimal τ for him to attack:

$$\frac{\pi(t_i+\tau|t_i)}{1-\Pi(t_i+\tau|t_i)} = \frac{c}{g(t_i+\tau-T^{*-1}(t_i+\tau))}. \tag{8}$$

We also check the second order condition, which turns out that the second order derivative is negative and the second order condition is satisfied.

Taking Equation (6) into the left hand side of the first order condition gives us:

$$\frac{\pi(t_i+\tau|t_i)}{1-\Pi(t_i+\tau|t_i)} = \frac{1}{\zeta-\tau}. \tag{9}$$

In addition, in this symmetric equilibrium, the duration between the time when the regime collapses and the time when the overvaluation happens is given by: $t_i+\tau-T^{*-1}(t_i+\tau) = \tau^*+k\eta = \zeta$. This is because each speculator will delay a period of τ^* and the regime will collapse exactly at the moment $t_0+k\eta+\tau^*$ when the speculator $t_0+k\eta$ launches his attack.

So we find:

$$\frac{1}{\tau^*+k\eta-\tau} = \frac{c}{g(\tau^*+k\eta)}. \tag{10}$$

Since it is a symmetric equilibrium, $\tau = \tau^*$. Solving the above equation, we get

$$\tau^* = \frac{(c-g)k\eta}{g}. \tag{11}$$

Given $\frac{c}{g}k\eta < \tau'$, the regime will collapse at $t_0 + k\eta + \tau^* < t_0 + \tau'$ endogenously.

Notice that $\tau^* \geq 0$ if and only if $c \geq g$. When $c < g$, we will get the corner solution of $\tau^* = 0$.

Q.E.D

The intuition of the equilibrium is as follows. Given that all the speculators begin their attack at $t_i + \tau^*$, the instantaneous probability that the regime collapses at $t_i + \tau$ of speculator t_i is given by:

$$\frac{\pi(t_i + \tau | t_i)}{1 - \Pi(t_i + \tau | t_i)} = \frac{1}{\tau^* + k\eta - \tau}. \tag{12}$$

If the regime exactly collapses at $t_i + \tau$, the gains from attacking will be $g(\tau^* + k\eta)$. Thus, the expected marginal benefits of speculator t_i attacking at $t_i + \tau$ are given by:

$$g(\tau^* + k\eta)\frac{\pi(t_i + \tau | t_i)}{1 - \Pi(t_i + \tau | t_i)} = g(\tau^* + k\eta)\frac{1}{\tau^* + k\eta - \tau}.$$

Meanwhile, the marginal costs incurred by attacking at time $t_i + \tau$ are c, which are constant. From the above equations we can see that the expected marginal gains from attacking are strictly increasing in τ, since the speculator t_i's subjective instantaneous probability that the regime collapses at time $t_i + \tau$ is strictly increasing in τ. So there is a unique level of τ, where the expected marginal gains from attacking are exactly equal to the marginal costs incurred by attacking. And it is the optimal time for speculator t_i to attack. Figures 2 and 3 explain the intuition.

3.3 Comparative Statics

This section studies how the changes in parameters of the model influence equilibrium results.

We know that in equilibrium

$$\tau^* = \frac{(c-g)k\eta}{g}.$$

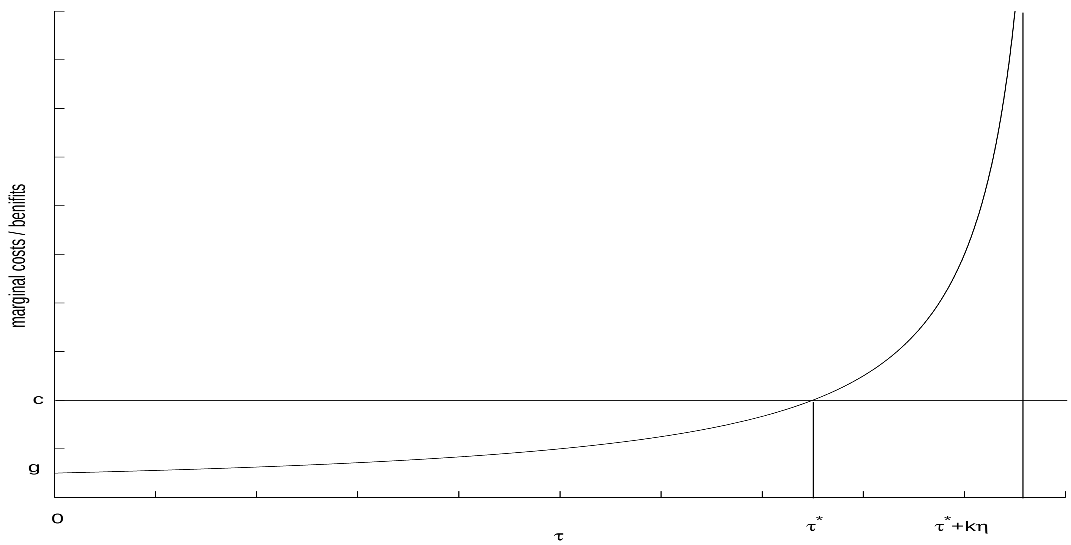

Figure 2: How the marginal costs and benefits change in τ in the case of the interior solution of τ^*

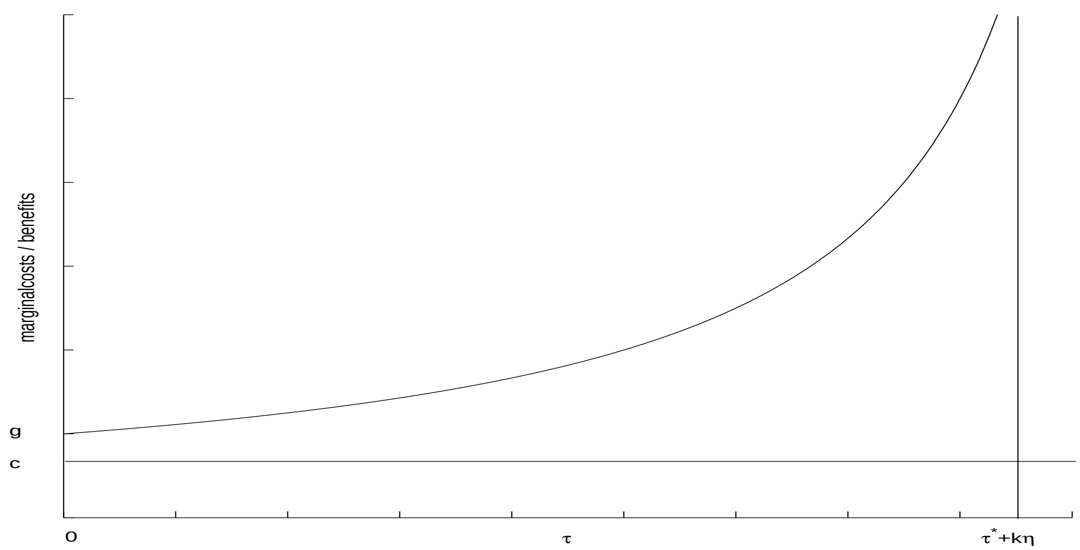

Figure 3: How the marginal costs and benefits change in τ in the case of the corner solution of τ^*

First, we can see that the speculators will wait longer with higher c. The intuition is simple. Higher c means that it will cost more if a speculator launches an attack early. Hence a speculator would like to wait longer to reduce the costs of attacking.

Second, we find that the speculators will wait longer with both higher k and η. This result is also intuitive. Higher η means more dispersed opinions among the speculators and higher k means a higher requirement for coordination. Both will increase the difficulties in coordination and induce the speculators to wait longer.

We know that c, k and η are all parameters indicating how difficult it is to arbitrage in a foreign exchange market. We find that now the frictions in the market become a blessing for the speculators, since more frictions will induce the speculators to wait longer and make higher profits from the overvaluation.

Third, we find that the speculators will wait longer with lower g, the rate at which the currency is overvalued. In this case, higher g increases the speculators' incentive to preempt other speculators and makes the speculators less patient. In the extreme case when $g > c$, speculators will launch an attack immediately after they become aware of the overvaluation.

Finally, there is an interesting result about the exchange rate level when the regime collapses, which determines the magnitude of the devaluation. It is given by $ck\eta$. We can see that g does not play a role in determining the magnitude of the devaluation. This is because the speed at which the fundamental value of the currency decreases has two opposite effects: First, it affects the optimal delay time of speculators. Second, it affects the fundamental exchange rate at time t. The net result from these two effects is that g will not influence the exchange rate when the regime collapses at all.

4 The Model with a Large Player

In this section we introduce a large player into the basic model.

We keep the model as simple as possible, by assuming that the speculators consist of one large player with wealth $\lambda < k$ and a continuum of small speculators of mass 1 with total wealth of 1. Here we assume $\lambda < k$ such that the large player cannot independently break the peg. This assumption is realistic because even a large player like Soros in financial markets cannot single-handedly break a currency peg. Moreover, we assume that the large player has perfect information about t_0;

that is, he always becomes aware of the overvaluation at t_0 when the overvaluation happens. In addition, we assume that the action of the large player will not be observed by other speculators.

Now we need to define the equilibrium in such a setup. Given all the assumptions unchanged for small speculators, we will prove that there is a unique trigger strategy equilibrium in this game.

Proposition 2. *Given $\tau' > \frac{(c-g)(k-\lambda)\eta}{g}$ and $\frac{c}{g} > \frac{k}{k-\lambda}$, there is a unique trigger strategy equilibrium where the regime collapses endogenously. In this equilibrium, each small speculator t_i begins to attack at the instant $t_i + \tau^{SP}$ and keeps attacking until the regime collapses. The large player begins to attack at $t_0 + (k-\lambda)\eta + \tau^{SP}$. Here $\tau^{SP} = \frac{(c-g)(k-\lambda)\eta}{g}$. The regime collapses exactly at $t_0 + \frac{c(k-\lambda)\eta}{g}$, when the large player launches the attack.*

Given $\tau' > \frac{c(k+\lambda)\eta}{g}$ and $\frac{k}{\lambda+k} < \frac{c}{g} < 1$, there is a unique trigger strategy equilibrium where the regime collapses endogenously. In this equilibrium, each small speculator begins to attack at the instant $t_i + \tau^{SP}$ and keeps attacking until the regime collapses. Here $\tau^{SP} = \frac{(c-g)k\eta + c\lambda\eta}{g}$. The large player begins to attack at $t_0 + k\eta + \tau^{SP}$. The regime collapses exactly at the time when the large player launches the attack.

Given $\tau' > k\eta$ and $\frac{c}{g} < \frac{k}{\lambda+k}$, there is a unique trigger strategy equilibrium where the regime collapses endogenously. In this equilibrium, each small speculator begins to attack at the instant t_i and keeps attacking until the regime collapses. The large player begins to attack at $t_0 + k\eta$. The regime collapses exactly at the time when the large player launches the attack.

Proof

Since the large player has perfect information about t_0, he will choose the optimal time $t_0 + \tau^{LP}$ to maximize his profits, given the equilibrium strategies taken by small players. Since small players are identical ex ante and atomically small, they will take symmetric strategies. Suppose that each small player plays the symmetric trading strategy $t_i + \tau^{SP}$ in equilibrium. From the moment of $t_0 + (k-\lambda)\eta$ on, the total wealth of the large player and small players exceeds the threshold level k. Thus, the payoffs of the large player from attacking at $t_0 + (k-\lambda)\eta + \tau^{SP} + t$ are given by

$$\lambda g[(k-\lambda)\eta + \tau^{SP} + t]\frac{k}{k+\frac{t}{\eta}} = \lambda g k\eta \frac{(k-\lambda)\eta + \tau^{SP} + t}{k\eta + t},$$

where $0 \leq t \leq \lambda\eta$.

Notice that $t \leq \lambda\eta$, or the regime will collapse solely due to the attacking pressure from small players, and the large player will gain zero. The large player will choose an optimal level of t to maximize his expected payoff. Solving the maximization problem, we get that $t = 0$ given $\lambda\eta - \tau^{SP} < 0$, and $t = \lambda\eta$ given $\lambda\eta - \tau^{SP} > 0$.

Therefore, the optimal strategy for the large player is as follows. Given $\lambda\eta - \tau^{SP} < 0$, the large player will launch the attack at $t_0 + \tau^{LP}$, where $\tau^{LP} = (k-\lambda)\eta + \tau^{SP}$. Given $\lambda\eta - \tau^{SP} > 0$, the large player will launch the attack at $t_0 + \tau^{LP}$, where $\tau^{LP} = k\eta + \tau^{SP}$. (The intuition for the above results is as follows. When the large player delays his attacking, there are two effects on his payoffs. First, he will gain more from the larger devaluation when the regime collapses. Second, he will gain less due to the smaller share in the total attacking wealth. The shorter τ^{SP} is, and the larger λ and η are, the more the large player will gain from delaying.)

Now let us look at the best responses of small players. Our previous proof for the unique symmetric trigger strategy equilibrium still holds in this case. Only now the optimal attacking time $t_i + \tau^{SP}$ is determined by the following conditions.

Given the optimal strategy of the large player, $\tau^{LP} = (k-\lambda)\eta + \tau^{SP}$, in equilibrium the regime collapses at $T^* = t_0 + \zeta = t_0 + (k-\lambda)\eta + \tau^{SP}$. Therefore, the first order condition gives

$$\frac{\pi(t_i + \tau^{SP}|t_i)}{1 - \Pi(t_i + \tau^{SP}|t_i)} = \frac{1}{\zeta - \tau^{SP}} = \frac{1}{(k-\lambda)\eta} = \frac{c}{g\zeta}.$$

In equilibrium, $\zeta = (k-\lambda)\eta + \tau^{SP}$. Thus we get $\tau^{SP} = \frac{(c-g)(k-\lambda)\eta}{g}$. The large player's equilibrium strategy is $\tau^{LP} = (k-\lambda)\eta + \tau^{SP} = \frac{c(k-\lambda)\eta}{g}$. Checking the condition inducing the large player to choose $\tau^{LP} = (k-\lambda)\eta + \tau^{SP}$, we get:

$$\lambda\eta - \tau^{SP} < 0 \Rightarrow \frac{c}{g} > \frac{k}{k-\lambda}.$$

Now let us look at the case in which the large player takes the equilibrium strategy of $\tau^{LP} = k\eta + \tau^{SP}$. In equilibrium $T^* = t_0 + \zeta = t_0 + k\eta + \tau^{SP}$. Given the large player's equilibrium strategy, the first order condition for small players is given by

$$\frac{1}{\zeta - \tau^{SP}} = \frac{c}{\frac{k}{k+\lambda} g(k\eta + \tau^{SP})} = \frac{1}{\eta k} = \frac{c(k+\lambda)}{kg(k\eta + \tau^{SP})}.$$

Solving the above equation, we get $\tau^{SP} = \frac{(c-g)k\eta + c\lambda\eta}{g}$. Therefore, $\tau^{LP} = k\eta + \tau^{SP} = \frac{c(k+\lambda)\eta}{g}$. We need to check the condition inducing the large player to choose $\tau^{LP} = k\eta + \tau^{SP}$, which is

$$\lambda\eta - \tau^{SP} > 0 \Rightarrow \frac{c}{g} < 1.$$

Moreover, notice that in order to ensure τ^{SP} is positive, we have $c(\lambda + k) - gk > 0$, or $\frac{c}{g} > \frac{k}{\lambda + k}$. When $\frac{c}{g} < \frac{k}{\lambda + k}$, we get the corner solution of $\tau^{SP} = 0$. Since $\lambda\eta - \tau^{SP} > 0$ in this case, the condition required for the large player to choose the strategy of $\tau^{LP} = k\eta + \tau^{SP}$ still holds. Thus, the general condition for the large player to take the strategy of $\tau^{LP} = k\eta + \tau^{SP}$ is $\frac{c}{g} < 1$.

Q.E.D.

5 The Role of a Large Player

In this section, we analyze the role that a large player plays in a currency attack. Our model reveals that a large player can both accelerate or delay the collapse of a fixed exchange rate regime, depending on the circumstances. This result is important because it differs from the usual perception that the presence of a large player in a foreign exchange market will facilitate arbitrage, therefore helping to reduce the mispricing of exchange rates.

From Proposition 6, we can see that there are two possible equilibria. We will analyze these two cases respectively.

5.1 The Case in Which a Large Player Accelerates the Attack

Given $\frac{c}{g} > \frac{k}{k-\lambda}$, the presence of the large player will accelerate the collapse of the regime. The following are some results we find in this case.

1. *The collapse of the regime is accelerated due to two reasons: first, the large player has perfect information about t_0. Thus, more speculators are aware of the mispricing from t_0 on. Second, the presence of a large player makes small players more aggressive and shortens their delay time.*

   The regime will collapse at $t_0 + \frac{c(k-\lambda)\eta}{g}$, which is earlier than the regime collapse time without a large player (which is $t_0 + \frac{c}{g}k\eta$). Here the collapse time is earlier, for two reasons: first, the large player will begin to attack exactly when there is enough wealth to correct the overvaluation. Thus, the regime collapses as long as mass of $k-\lambda$ of small players attacks, instead of mass of k in the case without a large player. Second, with the presence of the large player, the small players' equilibrium strategy, which is the waiting time between becoming aware of the overvaluation and before starting an attack, is shorter. In the case without a large player, the small players' strategy is $\tau^* = \frac{c-g}{g}k\eta$. With the large player it becomes $\tau^{SP} = \frac{(c-g)(k-\lambda)}{g}\eta$. In this case, the presence of a large player makes small players take more aggressive strategies and accelerates the arbitrage to correct the overvaluation.

2. *The collapsing time is strictly decreasing in λ, and the devaluation will be also smaller at the collapse time with larger λ. However, λ must be low enough to ensure the existence of this equilibrium.*

   Since $t_0 + \frac{c(k-\lambda)\eta}{g}$, the more wealth a large player has, the faster the fixed exchange rate regime will collapse. The exchange rate at the collapse time is given by $E_0(1 + c(k-\lambda)\eta)$, which is also decreasing in λ. However, in order for this accelerating equilibrium to exist, we must have $\frac{c}{g} > \frac{k}{k-\lambda}$. That is, $\lambda < \frac{(c-g)k}{c}$. Thus, this equilibrium will exist only when the wealth of the large player is low enough.

3. *The large player can make the most profits from the attack when $\lambda = \frac{k}{2}$.*

   The profits of the large player are given by:

$$\lambda g \frac{c(k-\lambda)\eta}{g}.$$

It is straightforward to see that the optimal λ to maximize the large player's payoffs is $\lambda = \frac{k}{2}$. So there is not a monotonically increasing relationship between the wealth of the large player and the payoffs it reaps from the attack. The intuition is that there is a tradeoff with the increase of the wealth of the large player. On the one hand, more wealth ensures that the large player can claim a higher proportion of the attacking wealth that profits from the collapse of the regime. On the other hand, higher wealth will accelerate the collapse of the regime, leading to less devaluation, and therefore lower profits when the regime collapses.

5.2 The Case in Which a Large Player Delays the Attack

Given $\frac{c}{g} < 1$, the presence of a large player delays or causes no acceleration of the regime collapse. The following are some results that we get in this case.

1. *The collapse of the regime is delayed or is not accelerated for two reasons: First, a large player chooses to "ride the overvaluation." Second, the presence of a large player makes small players less aggressive and wait longer before launching the attack.*

   Given $\frac{k}{\lambda+k} < \frac{c}{g} < 1$, the regime will collapse at $t_0 + k\eta + \tau^{SP}$, where $\tau^{SP} = \frac{(c-g)k\eta + c\lambda\eta}{g} > 0$. However, in the case without a large player, we get the corner solution of $\tau^* = 0$, which we can interpret as being that the speculators will launch an attack as soon as they become aware of the overvaluation. There are two reasons to explain why small players delay their attack. First, small players are aware that the large player will "ride the overvaluation." Second, due to the presence of the large player, the gains of small players from the attack will be less, which reduces the incentive of small players to preempt other players. In order to see this, recall that the equilibrium equation to determine τ^{SP} is given by:

$$\frac{1}{\eta k} = \frac{c}{\frac{k}{k+\lambda} g(k\eta + \tau^{SP})},$$

which is slightly different from that in the case without a large player:

$$\frac{1}{\eta k} = \frac{c}{g(k\eta + \tau^*)}.$$

The only difference between the above two equations is that with the presence of a large player, the expected payoffs of a small player will be the proportion of $\frac{k}{\lambda+k}$ of the total devaluation, instead of the whole devaluation.

Given $\frac{c}{g} < \frac{k}{\lambda+k}$, the presence of a large player will at least cause no acceleration of the collapse of the regime. In this case we get the corner solution of $\tau^{SP} = 0$. Thus the collapse time of the regime will be $t_0 + k\eta$, which is the same as the case without a large player.

This result differs from our common belief that the presence of large players facilitates the arbitrage and alleviates the mispricing in foreign exchange markets. The intuition here is that the market power of the large players, due to their superior information and more wealth, gives them the ability to time the collapse of the regime and " ride the overvaluation." In certain circumstances they prefer to wait longer to reap the most profits from the currency overvaluation.

2. *The collapse time of the regime is strictly increasing in λ. The devaluation will also be greater when the regime collapses with larger λ.*

   Given $\frac{k}{\lambda+k} < \frac{c}{g} < 1$, the collapse time is given by $t_0 + \frac{c(k+\lambda)\eta}{g}$, which is strictly increasing in λ. Moveover, the exchange rate at the collapse of the regime is given by $E_0(1 + c(k + \lambda)\eta)$, which is also strictly increasing in λ. Therefore, the more wealth the large player has, the later the regime will collapse, and the larger the devaluation will be at the time of the collapse. Notice that in order for $\tau^{SP} > 0$, $\lambda > \frac{(g-c)k}{c}$. So in this equilibrium λ has to be large enough to induce small speculators to wait some time after being aware of the overvaluation.

3. *The profits of the large player are strictly increasing in λ.*

   This is straightforward to see from the payoff function of the large player:

$$\lambda g \frac{c(k+\lambda)\eta}{g} = c\lambda(k+\lambda),$$

which is strictly increasing in λ.

Our model reveals that the ratio of $\frac{c}{g}$ is critical to determine whether the presence of a large player will accelerate or delay a currency attack. When $\frac{c}{g} > \frac{k}{k-\lambda} > 1$, the presence of a large player will accelerate the attack. When $\frac{c}{g} < 1$, the presence of a large player will delay or at least will not accelerate the attack.

The intuition is as follows. g and c are key to determining τ^{SP}, that is, how long small players will wait before launching an attack. Moveover, τ^{SP} is critical to determining the gains a large player will get from delay, relative to the losses from delay. The shorter τ^{SP} is, the more are the gains relative to the losses. Thus, higher g and lower c lead to shorter τ^{SP}, inducing the large player to choose the delay equilibrium. Meanwhile, lower g and higher c lead to longer τ^{SP}, inducing the large player to choose the accelerating equilibrium. In summary, only when g is large enough and c is low enough, is the incentive of a large player to "ride the overvaluation" strong enough to dominates the incentive to preempt small speculators, and therefore to make him wait longer. Therefore, his presence delays the regime collapse, and leads to severe currency overvaluation in the attacked country.

Here we argue that the presence of a large player will be harmful to a small economy in the sense that a large player will employ his market power in a small economy to maximize his profits at the expense of the small economy. His presence prevents the small economy from correcting its overvalued currency in time and, we infer, causes more fluctuations in an economy once the devaluation happens.

6 A Note on the Application to the Stock Market

The basic results in our model can be extended to the stock market. Suppose that we introduce a large player into the stock market in the Abreu-Brunnermeier (2003) model. In their model, a continuum of small speculators have to decide the optimal time to sell their gradually overvalued stocks. Similar to the arguments we have developed above, we can introduce a large player who has perfect information about the time when the stock becomes overvalued. Given that small players will take a symmetric trigger strategy $t_i + \tau^{SP}$, the large player will choose his optimal strategy

$t_0 + \tau^{LP}$ to maximize his payoffs and vice versa. Here the large player also has the incentive to ride the bubble to maximize his payoffs from the burst of the bubble. So in principle, the presence of the large player can also delay the bursting of the bubble. Further analysis of the model is needed to obtain specific conditions under which the incentive of the large player to ride the bubble will lead to the delay from bursting the bubble.

7 Conclusions and Future Research

In this paper we study the role that large players play in currency attacks in a dynamic currency attack game where speculators have to determine when to attack, based on their incentives both to "ride the overvaluation" and to preempt other speculators. Our main finding is that a large player can accelerate or delay the collapse of a fixed exchange rate regime, depending on which incentive is dominant. More specifically, we find that when the incentive of a large player to "ride the currency overvaluation" dominates the incentive to preempt other speculators, the presence of a large player will delay the collapse of a fixed exchange rate regime. This finding is especially interesting because it differs from the common belief that the presence of large players will facilitate arbitrage and reduce asset mispricing.

One direction in which to extend the current model is to introduce multiple large players and to examine how equilibrium outcomes will change. In addition, in the current model with a large player, we assume the extreme case that the large player has perfect information about the time when the currency overvaluation begins. We can relax this assumption to a more general case where the large player has imperfect information about the time when the currency overvaluation begins.

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