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Credit Risk Transfer: To Sell or to Insure

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Abstract

This paper analyzes credit risk transfer in banking. Specifically, we model loan sales and loan insurance (e.g. credit default swaps) as the two instruments of risk transfer. Recent empirical evidence suggests that the adverse selection problem is as relevant in loan insurance as it is in loan sales. Contrary to previous literature, this paper allows for informational asymmetries in both markets. We show how credit risk transfer can achieve optimal investment and minimize the social costs associated with excess risk taking by a bank. Furthermore, we find that no separation of loan types can occur in equilibrium. Our results show that a well capitalized bank will tend to use loan insurance regardless of loan quality in the presence of moral hazard and relationship banking costs of loan sales. Finally, we show that a poorly capitalized bank may be forced into the loan sales market, even in the presence of possibly significant relationship and moral hazard costs that can depress the selling price.

Keywords: credit risk transfer, banking, loan sales, loan insurance

JEL Classification Numbers: G21, G22, D82.

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1 Introduction

The growth in credit risk transfer (CRT), and specifically, credit derivatives since the mid-90s has been large. Instruments such as bank loans, once virtually illiquid, can now have their risk stripped down and traded away. Indeed, how we view the role of banking institutions is fundamentally changing.

The growth in credit derivatives is illustrated in figure 1, where we see that the notional outstanding value has surpassed $12 trillion. Figure 2 is based on a survey of some of the largest financial institutions in the world. The average weekly trading volume for various derivative instruments is reported. We see that credit derivatives have overtaken "plain vanilla" equity derivatives in options activity for banks.

[Figure 1 about here]

[Figure 2 about here]

Duffee and Zhou (2001) gave us our first insight into how credit derivatives and loan sales can coexist.[1] The authors show how credit derivatives can help alleviate the "lemons" problem that plagues the loan sales market and that it is possible that the introduction of credit derivatives could shut down the loan sales market. This paper builds on Duffee and Zhou (2001), but departs from it in two important ways. First, an assumption that is pivotal to their lemons result is that loan insurance is used when no informational asymmetries exist between the bank and the potential insurer. Recent empirical evidence by Acharya et al. (2005) suggests that banks are acting on their privileged information in credit default swaps (loan insurance) markets. In their analysis, they find significant information is revealed within these derivatives markets. This information revelation is a tell-tale sign that banks are trading with asymmetric information that can give rise to adverse selection. The first contribution of this paper is to extend the Duffee and Zhou framework by

[1] A loan sale trades in the same way as the sale of any other type of asset: When a loan is sold, the future income stream as well as all default risk is taken off the sellers books (note that we are not considering a situation where the bank can make a contractual guarantee about the loan's outcome, namely, we consider only loan sales without recourse). Alternatively, in a loan insurance contract, the risk buyer agrees to cover the losses that take place if pre-defined events happen to the underlying firm. (In many cases, this event is the default of the underlying loan. However, some contracts also include things like re-structuring as a triggering event). In exchange for this protection, the risk shedder agrees to pay an ongoing premium. Therefore, the credit risk of the underlying loan is transferred from the risk-shedder's books, but the ownership of the loan still remains with its originator. The instrument we refer to as loan insurance in this paper most closely resembles a credit default swap contract. As of mid-2005, single-name credit default swaps accounted for two-thirds of all gross sold credit derivative positions (Fitch 2005).

allowing for informational asymmetries in the credit default swap market. Second, Duffee and Zhou (2001) assume that loan insurance is written on the first period of a two period loan. This assumption is restrictive too, because it implies a maturity mismatch.[2] The Basel Committee on Banking Supervision (2005) found that supervisors penalize banks in terms of regulatory capital if there is a maturity mismatch. There are even cases where this practice would yield no regulatory capital relief at all. The new Basel II agreement formalizes what most supervisors are currently doing by only allowing maturity mismatches in some cases, but reducing the regulatory capital benefit of the hedge in those instances (BIS 2005). Therefore, we analyze the consequences for the credit derivatives (and sales) market when the insurance contract has the same maturity as the underlying loan.[3] The predictions of our model are significantly different than Duffee and Zhou (2001) and will be discussed below.

In this paper we look at *how* risk is disseminated in the banking sector. This sets this paper apart from the work of others,[4] who assume a structure of credit risk transfer, and analyze the consequences for issues such as market liquidity and financial stability. We begin by putting structure on the asymmetric information problem so that we can price our instruments.[5] First, through the unique relationship with their borrower, the bank may learn that a loan is of poor quality. Second, there may be another investment available, which, when combined with the original loan, may create a risk level that is unpalatable for the bank. We show how both loan sales and credit derivatives can be used to achieve optimal levels of investment, while minimizing undue banking risk. We seek to differentiate the two products within the banking environment by concisely determining under what conditions one is advantageous to the other, and when each can be sustained in an equilibrium setting. We show that in equilibrium, no separation of types can occur. We find that two pooling equilibria can exist: one insurance and one sales. Determining when each pooling equilibrium is unique, we find that well capitalized banks will wish to exclusively use loan insurance. Alternatively, banks who must utilize costly capital may need to turn to the loan sales market, even when there are relationship management[6] and moral hazard concerns that can depress

[2] A maturity mismatch introduces an example of an additional risk referred to as basis risk.

[3] Although the maturity of the contract is the same, the loan still belongs to the bank. The no maturity mismatch rule we introduce simply allows the bank to trade all of the credit risk away, instead of just a timed portion of it.

[4] Excluding Duffee and Zhou (2001).

[5] This was not an issue in the Duffee and Zhou (2001) framework since the absence of asymmetric information in the credit derivatives market yielded fully informed pricing.

[6] Relationship management refers to the unique bank-borrower relationship that is established through the course

the selling price. By introducing these features of loan sales and loan insurance, our results differ significantly from those of Duffee and Zhou (2001). Here, loan insurance is in direct competition with loan sales, and the result is that it may or may not be optimal to use, depending on the new factors we introduce. The fact that uniqueness of the two possible pooling equilibria can be determined by the relative severity of costly capital to moral hazard and relationship management costs constitutes the new predictions of our model, and the main contribution of this paper.

The intuition of our results is as follows. When the bank needs to reduce the risk in its portfolio, it can decide whether to use loan sales or loan insurance. The risk buyer then prices these contracts given the available information. If the perceived probability of default from the risk buyers' perspective is the same for both instruments, then a bank with a good loan would prefer to use insurance, since it need only secure their initial investment, and the return remains solely the bank's. However, banks with bad loans have no incentive to truthfully reveal the quality of their loan by using sales. Therefore, only a pooling insurance, or pooling sales equilibrium can exist. We are left with identifying which pooling equilibrium will prevail. To do this, we extend the previous analysis to the more realistic case in which capital is costly, and where moral hazard and relationship banking issues are present in the bank-firm relationship. In this case, the results get more complicated, but the intuition is clear. In loan sales, the bank will have little incentive to continue to monitor the loan after a sale and could also lose some of the relationship it has built with the underlying firm. In loan insurance, ties are maintained to the underlying loan so the incentive to continue monitoring is greater. As well, the firm need not know that an insurance contract was signed, so the relationship is affected little. It is easy to see that if the relationship management or moral hazard problems are severe, then the bank will have an incentive to use insurance regardless of the loan quality. However, since insurance requires an upfront premium from the bank, whereas sales does not, costly capital works in the opposite direction. If capital is particularly costly for the bank, it may be optimal for the bank to use sales, regardless of loan quality.

The literature on credit risk transfer is small, but is growing. Gorton and Pennachhi (1995) provide an early and fundamental discussion of the moral hazard that can arise in CRT. With loan sales being the only instrument available in the model, they show how a bank can overcome the moral hazard problem by continuing to hold a fraction of the loan, and offering explicit guarantees

of a loan or loans.

on loan performance. In this setting, the incentive of the bank to continue monitoring the firm remains after the loan is sold. Duffee and Zhou (2001) extend the work of Gorton and Pennachhi (1995) to analyze the consequences of introducing credit derivatives as an instrument of risk transfer. This paper is most closely related to ours, in that they analyze loan sales and credit derivatives together.

In recent work, Parlour and Plantin (2006) analyze credit risk transfer through the bank-borrower relationship. Specifically, they use loan sales as their instrument of CRT and generate the adverse selection problem by a bank that has a stochastic discount shock and can exploit proprietary information. They analyze the case when a liquid CRT market can arise, and the socially inefficient outcome that may result. In contrast to their paper, we abstract away from the bank-borrower relationship and focus instead on the bank-risk buyer interaction. Furthermore, whereas Parlour and Plantin (2006) are concerned with when a liquid CRT market can arise, we restrict ourselves to the parameter values for which it exists. We do this because we are interested in the effects of both sales and insurance CRT markets together, and not the existence of a single market. Wagner and Marsh (2005) and Allen and Carletti (2006) model CRT in terms of loan sales to outside the banking sector. Wagner and Marsh (2005) study the social impact of CRT analyzing cases where CRT itself may not be efficient. They argue that setting regulatory standards that reflect the different social costs of instability in the bank and insurance sector will be welfare improving. Allen and Carletti (2006) show how a default by an insurance company can cascade into the banking sector causing a contagion effect when these two parties are linked through credit derivatives. What sets our paper apart is that these works are only interested in the result of a CRT contract, but not in the contract itself.

The paper proceeds as follows. Section 2 outlines the model. Section 3 analyzes the model in the absence of CRT. Section 4 analyzes CRT in the base case with no externalities. Section 5 extends the previous section on CRT to cases in which there is moral hazard, relationship management and costly capital present. Finally, in Section 6 we conclude. The appendix can be found in Section 7 where the proofs to a number of propositions are contained.

2 The Model Setup

The model shares the following features with Duffee and Zhou (2001): There are three dates, indexed as $t = 0, 1, 2$. There are three types of agents: a bank, a risk-taking counter parties (which we will refer to as risk buyers) behaving competitively and a firm (or entrepreneur) requiring capital for a project. The risk buying counter-party is risk neutral, while the bank, although maximizing a linear profit function will display risk aversion through an exogenous "regulation" parameter B to be explained below. The firm will be modelled simply as a production technology that can generate a fixed return or fail.

At time $t = 0$, the firm (entrepreneur) requests L_0 units of capital that yields a rate of return to the bank of $R_0 > 1$ if the firm's project succeeds at time $t = 2$. The bank then chooses $L \leq L_0$; we will discuss this choice further in section 3. The project is worth nothing if terminated at the interim period, $t = 1$. There are two types of projects: high quality and low quality. We assume there are half of each type of project in the economy, and a bank is assigned randomly to a project at time $t = 0$. Define p_h (p_l) as the probability that a high (low) quality project defaults (and returns zero), with $1 > p_l > p_h$. We assume that the bank privately learns the quality of the project at time $t = 1$. Without loss of generality, we normalize the risk free rate in the economy to zero. We also assume that the projects have positive net present value (NPV) so that it makes sense that the bank would take on such a loan. The bank is endowed with sufficient costless capital to undergo all desired investments. We will depart from the Duffee and Zhou (2001) setup and analyze the case of costly capital in section 5.2.

We add a new feature that Duffee and Zhou (2001) did not pursue by putting structure on the adverse selection problem. This departure is needed so that the prices can differentiate the two instruments to be introduced below. With no adverse selection in credit derivatives in Duffee and Zhou (2001), this structure was not needed. Equivalent to Parlour and Plantin (2006), we add a new investment opportunity that becomes available to the bank with probability q at time $t = 1$ that is private information to the bank. This investment has a return $R_1 > 1$ at time $t = 2$ if it succeeds but returns nothing with probability p_N. L_1 is required to be invested to pursue this new project.[7] This investment represents the dynamic nature of banking. The bank does not know

[7]Note that we can generalize this new investment to any concept that would make a bank wish to engage in CRT regardless of loan quality. For instance, Parlour and Plantin (2006) use a private stochastic bank discount factor.

what new opportunities will arise in the future when a loan is issued now. The fact that the risk buyer cannot observe whether this new investment was available is not crucial to our results. We discuss this further in section 4.

The Basel Committee on Banking Supervision (2005) cites two main reasons for the use of CRT by banks: The first is to free up credit to take on new business, while the second is to reduce risk due to capital requirements. Both of these points are captured by the two reasons a bank uses CRT in our model: Either they learn there is a new investment opportunity, and they need to shed loan risk to pursue it (to be described in greater detail below), or they are exploiting private information about the quality of the loan.

There is ample evidence that maintaining capital reserves is an important factor in banks' decisions to engage in CRT. Pennacchi (1988) provides the argument that a prime incentive for loan sales is to boost a bank's capital ratio. Dahiya et al. (2003) find empirically that most banks that engage in CRT fall into the bottom quartile when ranked against all banks by tier 1 capital.[8] Cebenoyan and Strahan (2002) find more supporting evidence of the capital motivation of CRT by directly showing that banks that sell their loans have less capital. To capture this capital consideration in a reduced form, we assume that the bank suffers a cost of $B > 0$ if its losses exceed some level, $\hat{L}$. We will address the interesting case in which $L_0 < \hat{L}$, $L_1 < \hat{L}$, but $L_0 + L_1 > \hat{L}$ (so a default of both loans causes this cost to be incurred). B is a loss that is unique to the banking environment. Because of the nature of their business, falling below certain levels of capital can be more costly for a bank than other types of institutions. We can interpret B in a couple ways. Consider B being associated with an event that causes fragility or even default of a bank.[9] As well, B could represent simply a regulatory penalty for a bank falling below a pre-determined level of capital. Justification for the loss parameter B can be found in a number of places.

Turning to the process of credit risk transfer, a risk buyer can insure the bank against losses in its original loan, or purchase that loan outright. Fitch (2005) reports that more than half of the credit derivatives traded remain in the banking system, with the next highest going to the insurance

[8]Tier 1 capital refers to financial capital considered to be the most reliable and liquid, equity being the most prevalent example.

[9]We could also think of a the banks excess risk taking is instability of the system as a whole. One could argue that the social cost to instability or failure in the banking sector is higher than it is with other types of financial institutions, because banks deal with the public, as in Wagner and Marsh (2005). Therefore, we could also think of B as a means for the bank to internalize the externality they are causing on the financial system.

system, and the third highest to hedge funds. Because of this observation, we simply model the risk buyer as any risk-neutral party, and give no characteristics that distinguish it as any one of the three key players on the risk buying side. The risk buyer does not learn the quality of the firm (while the bank does), and does not learn whether the new investment is available to the bank, but knows they appear with probability q (alternatively, the risk buyer sees the new investment, but is not able to determine if it is profitable for the bank; rather, they have a prior belief that with probability q, it is profitable). Note that the firm only enters this model through a project that needs funding. We will assume that the quality of the firm is only deduced through the bank-firm relationship; therefore, whether the firm knows its type or not is irrelevant. We present the timing of the model in the following figure.

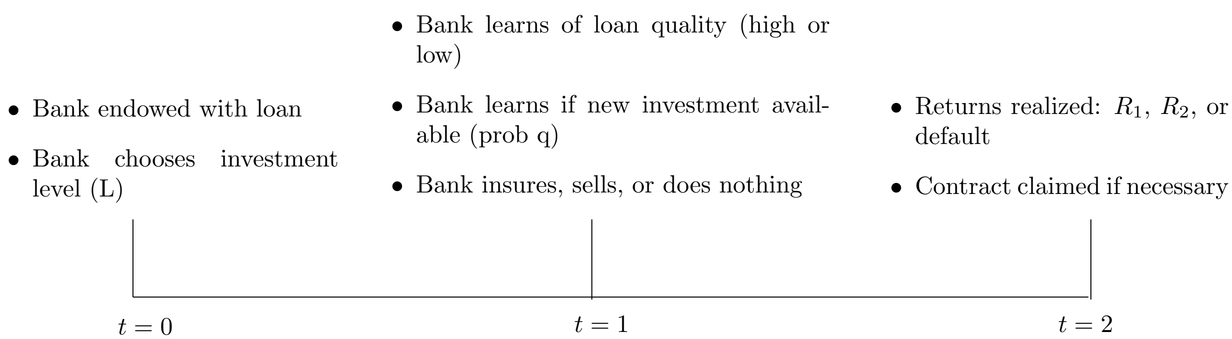

Figure 3: The timing of the Model

3 No CRT Available

We start with the benchmark case in which there are no CRT markets available. The following lemma allows us to pinpoint what initial investment levels the bank will wish to pursue. Specifically, the firm requests an investment of size L_0 and the bank chooses an investment level $L \leq L_0$. For simplicity, we will assume that if the initial investment is less than L_0, the project still proceeds with a return of $R_0 > 1$.[10] We will call the case in which $L = L_0$ *full (initial) investment*. Accordingly,

[10]Our assumption of constant returns to scale is not restrictive. We could modify this to be decreasing returns to scale so that the marginal return may go up when the project is underfunded. However, total profit, and therefore, total payment to the bank will go down. This is all that is required to get the results in the paper. Of course, assuming increasing returns to scale would reinforce our results even more.

$L < L_0$ will be called *under-investment*. We begin with the following lemma that gives the optimal investment strategy at time $t = 1$.

Lemma 1 *Suppose the new investment is available at $t = 1$. The bank's optimal investment strategy at $t = 1$ is characterized as follows: (i) if full initial investment is pursued at $t = 0$, then he bank will pursue the new investment when $B \leq \frac{R_1 L_1 (1-p_N)}{p_N p_h}$ ($B \leq \frac{R_1 L_1 (1-p_N)}{p_N p_l}$) when the loan is revealed as the high (low) type. (ii) If there is initial under-investment, the bank will always pursue the new investment.*

Proof. *See appendix.*

Interpreting the condition from this lemma is relatively straightforward. The numerator represents the expected value of the new investment, conditional on it being available. We see that the lower the probability of default of the new investment, the higher B can be and still maintain incentive to pursue it (for a fixed R_1 and L_1). We also see that the inequality is decreasing in the probability of default of the high or low quality original loan. Alternatively, we can rearrange the condition to $p_j \leq \frac{R_1 L_1 (1-p_N)}{p_N B}$, $j = \{h, l\}$ and the interpretation is simply that the new investment is pursued if and only if the old one is sufficiently safe. We now turn to time $t = 0$ and find the optimal investment strategy.

Lemma 2

In equilibrium, the bank chooses L_0 or $\hat{L} - L_1$ at $t = 0$.[11]

1. *If $B \leq \frac{R_1 L_1 (1-p_N)}{p_N p_l}$, the bank will set $L = L_0$ when $L - (\hat{L} - L_1) \geq \frac{q p_N B (p_h + p_l)}{R_0[(1-p_h)+(1-p_l)]}$, and $L = \hat{L} - L_1$ otherwise.*

2. *If $B > \frac{R_1 L_1 (1-p_N)}{p_N p_h}$, the bank will set $L = L_0$ when $L_0 - (\hat{L} - L_1) \geq \frac{2q(1-p_N) L_1 R_1}{R_0[(1-p_h)+(1-p_l)]}$, and $L = \hat{L} - L_1$ otherwise.*

3. *If $B \in (\frac{R_1 L_1 (1-p_N)}{p_N p_l}, \frac{R_1 L_1 (1-p_N)}{p_N p_h}]$, the bank will set $L = L_0$ when $L_0 - (\hat{L} - L_1) \geq \frac{q(1-p_N) L_1 R_1 - B p_N p_l}{R_0[(1-p_h)+(1-p_l)]}$, and $L = \hat{L} - L_1$ otherwise.*

[11] In this second case, L is simply the level of initial investment such that, when combined with the capital needed for the new investment, its maximum loses cannot exceed $\hat{L}$.

Proof. *See appendix.*

The first part of lemma 2 follows from the discontinuity in the payoff function over investment choices.[12] To interpret the second part of the lemma, we can re-write the condition in a more intuitive way:

$$R_0(L_0 - (\hat{L} - L_1))[\frac{1}{2}(1-p_h) + \frac{1}{2}(1-p_l)] \quad \geq \quad Bqp_N(p_h + p_l)$$

The left hand side is the return that the bank would forgo if it reduced its initial investment from L_0 to $\hat{L} - L_1$. The right hand side is the expected amount the bank would lose if it pursued full-investment. If $L_0 - (\hat{L} - L_1)$ is small, this means that the initial investment need not be reduced by much to avoid the cost of B. In this case, as long as B is not *too* big, the bank will find it advantageous to under-invest. Alternatively, when the left hand side is large, this means that the bank must reduce its investment by a lot to avoid the cost of B. In this case, unless B is very small, the bank will wish to invest fully. The third and fourth parts of the proposition can be re-arranged and interpreted in a similar fashion.

We now look at the base case where CRT is available to the bank. We do this so that we can enrich the model in section 5 to obtain our main results.

4 CRT Available

We now consider the case in which the bank pursues the initial investment fully, and can decide whether to pursue the new investment. With the availability of credit risk transfer markets, the risk buyer must price the loan sale or insurance premium, given the available information. The bank will wish to engage in credit risk transfer (CRT) if either it learns that the loan is of low quality, or the new investment becomes available. This can be assured by an assumption on B that will derived and discussed in section 4.1. Given the available information, the risk buyer can

[12] We can generalize this by making B a decreasing function of L_0. However, this would yield no further intuition about our problem, thus we use the simplest setup possible.

deduce the probability that a loan is of high quality (h) or low quality (l):

$$Prob(h|CRT) = \frac{q}{q+1}$$

The risk buyer can now form a belief about the probability that the loan will default:

$$Prob(Default|CRT) = \frac{p_l + qp_h}{q+1}$$

We allow the bank to insure its initial investment, or sell the loan outright. Because of their zero profit condition (competitiveness assumption), the risk buyer must be indifferent between insuring and not insuring, as well as selling and not selling. We assume that the bank insures its initial investment L,[13] and therefore, the risk buyer will demand a premium of $L_0(\frac{p_l+qp_h}{q+1})$. As well, the risk buyer would be willing to buy this loan for $R_0L_0(1-\frac{p_l+qp_h}{q+1})$. The latter is simply the expected payoff of the loan from the risk buyer's perspective.

If we relax the assumption that the risk buyer cannot observe the bank engaging in the new investment, we see that the adverse selection problem will still be present so long as the new investment is available. In this case, the risk buyer will use its prior beliefs to determine the probability of default. In the case where the new investment is not available, the risk buyer will know that the loan must be bad. However, in this paper, we are interested in the consequences for the insurance and sales market when adverse selection is present, so we rule out this revealing case by having the new investment be private information to the bank. Alternatively, we could assume that the new investment is public information but it is always available and all the results to be discussed would follow through.

4.1 Incentives to engage in CRT

In the previous section we outlined how the risk buyer would price a loan sale or insurance premium under the information structure given. We now outline restrictions on the parameter

[13] This assumption parallels that of Duffee and Zhou (2001) where the bank insures only its initial investment. If we allow the bank to insure less of their loan, i.e. partial insurance, this will not have any effects on the qualitative results of the paper, so long as the adverse selection problem is maintained. In other words, if the high type can reveal itself by insuring less of the loan, and still be protected from the cost B, the adverse selection problem would be solved. Since we have highlighted evidence that adverse selection is present in these markets, this paper will focus only this case.

space that will see the bank using CRT in the correct states. We assume that the bank invests the entire amount requested (L_0) in the initial investment. In Proposition 1 we will confirm that this level of investment will prevail in the presence of a CRT market. We begin by analyzing the incentives to insure, and then repeat the exercise for sales.

4.1.1 Incentives to Buy Insurance

We start by verifying that the bank will wish to purchase loan insurance in the appropriate states. We denote the state where a high (low) quality firm is realized as H (L), and the state where the new investment opportunity is realized (not realized) as NEW (NONEW). Therefore {H,NEW} represents a high quality firm and a new investment available.

It is easy to show that if the incentives are such that the bank insures in the high state, then this implies they will insure in the low state. We therefore need to check two states: {H,NEW} to make sure they wish to insure, and {H,NONEW} to make sure they do not wish to insure. Let us analyze {H,NONEW} first. Let π_{NI} denote the bank's payoff from no insurance in the high state, π_I denote the payoff from insurance in the high state, and P_I denote the price per unit of the insurance contract. We can consider the price P_I to be the insurance premium.

$$\pi_{NI} = R_0(1-p_h)L_0$$
$$\pi_I = R_0(1-p_h)L_0 - P_I L_0 + p_h L_0$$

From the above, for $\pi_{NI} \geq \pi_I$, $P_I \geq p_h$ must hold. This condition will be a constraint in the optimal contracting problem. We will refer to this condition as (I-Bound). We will assume that this condition holds by putting it as a restriction in the optimal contracting problem to be set out in section 4.3.

We now analyze {H,NEW} to see under what condition they will use loan insurance.

$$\pi_{NI} = R_0(1-p_h)L_0 + (1-p_N)L_1R_1 - p_h p_N B$$
$$\pi_I = R_0(1-p_h)L_0 - (P_I)L_0 + p_h L_0 + (1-p_N)L_1R_1$$

For $\pi_I \geq \pi_{NI}$ the following condition must hold:

$$B \geq \frac{L_0(P_I - p_h)}{p_h p_N} \tag{1}$$

Since $P_I \geq p_h$, the R.H.S of (1) is positive, and therefore we place this restriction on B.

4.1.2 Incentives to Sell the Loan

We can conduct a similar exercise for loan sales. We begin by analyzing {H,NONEW}. Let π_{NS} denote the payoff from no loan sales, and P_S be the price per unit of a sales contract with a net return on the underlying loan of 1.[14]

$$\pi_{NS} = R_0(1 - p_h)L_0$$
$$\pi_S = R_0(P_S)L_0$$

For $\pi_{NS} \geq \pi_S$, $P_S \leq 1 - p_h$ must hold. As in the insurance case, this condition will be a constraint in the optimal contracting problem. We will refer to this condition as (S-Bound).

We now analyze {H,NEW} to see under what condition they will use loan sales.

$$\pi_{NS} = R_0(1 - p_h)L_0 + (1 - p_N)L_1R_1 - p_N p_h B$$
$$\pi_S = R_0(P_S)L_0 + (1 - p_N)L_1R_1$$

For $\pi_S \geq \pi_{NS}$ the following must hold:

$$B \geq \frac{R_0 L_0(1 - p_h - P_S)}{p_h p_N} \tag{2}$$

Therefore, (2) is the parametrization that we make for loan sales. If we consider each market in isolation, we get $P_I = Prob(Default|insurance) = Prob(Default|sales) = 1 - P_S$. Therefore, one can see that the only difference between (1) and (2) is a factor of R_0. The reason for this difference is intuitive when we look at how the two contracts are priced. Whereas the insurance premium

[14] P_S is defined in this way to ease the comparison of loan sales to loan insurance. Therefore, the total price of the loan sale contract is $R_0 L_0 P_S$.

is independent of the return on the initial investment, the sales contract involves an entitlement to the return in the future, and must depend on R_0. If R_0 is very high, B must also be high so that in {H,NEW} the bank still has an incentive to use CRT. Therefore, we make the following assumption so that CRT can arise.

Assumption 1 *To permit the existence of both CRT markets, we let* $B \geq \max\{\frac{L_0(P_I - p_h)}{p_h p_N}, \frac{R_0 L_0(1-p_h-P_S)}{p_h p_N}\}$.

The form of this assumption deserves some explanation. We allow the assumption to depend on our two endogenous variables (P_I and P_S) because we are looking at the consequences of the two CRT markets. In other words, we assume the existence of a CRT market for the equilibrium price in which the risk buyer will earn zero profit. The parameter space can always be chosen to accomplish this.

4.2 CRT Available - Either Loan Sales or Loan Insurance (but not both)

Below, we will show the ex-ante expected payoff equivalence of *loan sales* and *loan insurance*. Because of this, we need only check one or the other to investigate under what conditions full investment can arise. The case in which the bank does not use CRT and opts to reduce risk exposure by under-investing is given in the appendix as equation (13). We can derive the expected (ex-ante) payoff if the bank uses CRT to reduce credit exposure:

$$\begin{aligned}
E(\pi_I) &= (\frac{1}{2})(1-q)[(1-p_l)R_0L_0 + p_lL_0 - L_0(\frac{p_l + qp_h}{q+1})] \\
&+ (\frac{1}{2})(q)[(1-p_l)R_0L_0 + p_lL_0 - L_0(\frac{p_l + qp_h}{q+1}) + (1-p_N)L_1R_1] \\
&+ (\frac{1}{2})(q)[(1-p_h)R_0L_0 + p_hL_0 - L_0(\frac{p_l + qp_h}{q+1}) + (1-p_N)L_1R_1] \\
&+ (\frac{1}{2})(1-q)[(1-p_h)R_0L_0] \\
&= \frac{1}{2}R_0L_0(1-p_h) + \frac{1}{2}R_0L_0(1-p_l) + q[(1-p_N)L_1R_1] \qquad (3)
\end{aligned}$$

Equation (3) shows us that the expected payoff to the bank is simply the expected return from the initial loan ($\frac{1}{2}R_0L_0(1-p_h) + \frac{1}{2}R_0L_0(1-p_l)$) plus the expected return from the new investment ($q[(1-p_N)L_1R_1]$). Similarly, denoting expected profit from loan sales by $E(\pi_S)$ we derive the following expression:

$$\begin{aligned}
E(\pi_S) &= (\frac{1}{2})(1-q)[R_0L_0(1-\frac{p_l+qp_h}{q+1})] \\
&+ (\frac{1}{2})(q)[L_0R_0(1-\frac{p_l+qp_h}{q+1}) + (1-p_N)L_1R_1] \\
&+ (\frac{1}{2})(q)[L_0R_0(1-\frac{p_l+qp_h}{q+1}) + (1-p_N)L_1R_1] \\
&+ (\frac{1}{2})(1-q)[(1-p_h)R_0L_0] \\
&= (\frac{1}{2})R_0L_0(1-p_h) + (\frac{1}{2})R_0L_0(1-p_l) + q[(1-p_N)L_1R_1] \qquad (4)
\end{aligned}$$

The equivalence of (3) and (4) has been established.

We are now equipped to show the main proposition of this section. The following proposition analyzes the use of CRT and its effect on investment choice.

Proposition 1 *CRT is ex-ante more profitable than without and yields the efficient level of initial and new investment.*

Proof. *See appendix.*

From lemmas 2 and 1, it is straight-forward to see that the ex-ante expected loss due to B can be eliminated by either investing in CRT in both states of the world, or under-investing.

4.3 Both markets available - Equilibrium

The equilibrium concept we apply is that of a Bayesian Nash equilibrium (BNE). Given Assumption 1, there are only two equilibria that can prevail when both CRT markets are open at the same time. The first equilibrium is where all types use loan insurance, while the second is where all types use loan sales. We first verify that a separating equilibrium cannot exist, and then proceed to see which pooling equilibria can be sustained.

4.3.1 Non-existence of Separating Equilibria

From proposition 1, we know that full initial investment dominates under-investment. For what follows, we will assume that the conditions of lemma 1 are satisfied so that the bank prefers to invest in the new investment when it is available. Checking both {L,NONEW} and {L,NEW} is redundant. First, the participation constraint on {H,NEW} is a stronger condition than either of the participation constraints for the low types. As well, the new investment will yield the same additional payoff in both sales and insurance, so the one incentive constraint is redundant.[15] Finally, assumption 1 guarantees that the bank will not wish to use CRT in the state {H,NONEW}. Thus, we need only check {L,NONEW} and {H,NEW}.

Consider first a separating equilibrium where a bank with type high (low) type loans chooses to insure (sell). Given the information structure of this separating equilibrium, we know that $Prob(Default|insurance) = p_h$ and $Prob(Default|sales) = p_l$. We can show that this equilibrium cannot exist simply by looking at the incentive constraint on the low type (IC1 - which says that the low type wishes to use sales over insurance) as well the two zero profit conditions. The optimal prices (P_I,P_S) must satisfy the following:

$$R_0L_0(P_S) \geq (1-p_l)R_0L_0 - L_0P_I + p_lL_0 \tag{IC1}$$

$$L_0(P_I - p_h) = 0 \tag{zero-π_I}$$

$$R_0L_0[(1-p_l) - P_S] = 0 \tag{zero-π_S}$$

By (zero-π_I) and (zero-π_S), we can see that the only candidate prices are $P_I = p_h$ and $P_S = 1 - p_l$. We can quickly verify that under these prices, (IC1) cannot hold:

$$(1-p_l)R_0L_0 \geq (1-p_l)R_0L_0 - L_0p_h + L_0p_l$$
$$\Rightarrow p_h \geq p_l$$

Since $p_l > p_h$, (IC1) is violated. Therefore, this separating equilibrium cannot be supported. We can also verify that the separating equilibrium where banks with high quality loans sell, and banks

[15]To see this, consider the participation constraints in the state {L,NEW}: $(1-p_l)R_0L_0 - L_0P_I + p_lL_0 + (1-p_N)L_1R_1 \geq (1-p_l)R_0L_0 + (1-p_N)L_1R_1 - p_lp_NB$ which simplifies to: $p_lL_0 - l_0P_I \leq p_lL_0 + p_lp_NB$. Now consider the participation constraint on {H,NEW}: $(1-p_h)R_0L_0 - L_0P_I + p_hL_0 + (1-p_N)L_1R_1 \geq R_0L_0(1-p_h) + (1-p_N)L_1R_1 - p_Np_hB$ which simplifies to: $p_hL_0 - l_0P_I \leq p_hL_0 + p_hp_NB$. Since $p_h < p_l$ it is obvious that if the participation constraint on {H,NEW} binds, the participation constraint on {L,NEW} will automatically bind. To see the redundancy in the incentive constraints, consider the incentive constraint where {L,NEW} wishes to use sales over insurance: $R_0L_0(P_S) + (1-p_N)L_1R_1 \geq (1-p_l)R_0L_0 - L_0P_I + p_lL_0 + (1-p_N)L_1R_1$. However, this simplifies to: $R_0L_0(P_S) \geq (1-p_l)R_0L_0 - L_0P_I + p_lL_0$ which is the incentive constraint on {L,NONEW}. This reasoning also applies if we are looking at the low type using insurance over sales. Therefore, only one of the incentive constraints on the low type is needed.

with low quality loans insure cannot be supported. The proof is very similar to the above and is omitted. The separating equilibria above are ruled out because the risk buyer is forced to earn zero profit in each market. Allowing them to subsidize one market by over-charging in the other will give rise to one separating equilibrium: a bank with high type loans will use insurance, and low type loans, sales. We pursue this case after we have finished analyzing the case in which the risk buyer must earn zero profit in each market. Any separating equilibrium where a type does not engage in CRT is ruled out by assumption 1.

4.3.2 Pooling Equilibrium with Insurance

We proceed by showing there are two possible pooling equilibria. Consider first the case where banks that have either high or low quality loans both choose to insure. Given the information structure in this pooling equilibrium, we know that $Prob(Default|insurance) = \frac{p_l+qp_h}{q+1}$. In this case we will have two participation constraints (I-PC1, I-PC2 - ensuring the bank with either the low or high type loans wishes to engage in CRT) and two incentive constraints (I-IC1, I-IC2 - ensuring that the bank with either the low or high type loans wish to use insurance over sales).[16] We can characterize the optimal prices (P_I,P_S) as follows:

$$(1-p_l)R_0L_0 - L_0P_I + p_lL_0 \geq (1-p_l)R_0L_0 \tag{I-PC1}$$

$$(1-p_h)R_0L_0 - L_0P_I + p_hL_0 + (1-p_N)L_1R_1 \geq R_0L_0(1-p_h) + (1-p_N)L_1R_1 - p_Np_hB \tag{I-PC2}$$

$$(1-p_l)R_0L_0 - L_0P_I + p_lL_0 \geq R_0L_0(P_S) \tag{I-IC1}$$

$$(1-p_h)R_0L_0 - L_0P_I + p_hL_0 + (1-p_N)L_1R_1 \geq R_0L_0(P_S) + (1-p_N)L_1R_1 \tag{I-IC2}$$

$$L_0(P_I - \frac{p_l+qp_h}{q+1}) = 0 \tag{zero-π}$$

$$P_I \geq p_h \tag{I-Bound}$$

$$P_S \leq 1-p_h \tag{S-Bound}$$

From (zero-π), we can see that the only admissible insurance premium is $P_I = \frac{p_l+qp_h}{q+1}$. At this price, (I-PC1) is satisfied, while (I-PC2) is satisfied by assumption 1.

[16] Recall that we need not check both the low type with and without the new investment as it is redundant.

From (I-IC1), we find:

$$P_S \leq (1 - p_l) + \frac{q(p_l - p_h)}{(q+1)R_0} \tag{5}$$

Next, from (I-IC2), we find:

$$P_S \leq (1 - p_h) - \frac{p_l - p_h}{(q+1)R_0} \tag{6}$$

Since $R_0 > 1$, $q \in (0, 1)$ and $p_l < p_h$, it follows that (5)$\Rightarrow$ (6). Therefore, (5) defines the price range that can be assigned to loan sales to sustain this pooling equilibrium, and represents the off-the-equilibrium path beliefs that the risk-buyer assigns to the sales market that can support this pooling equilibrium.[17] It is easy to see that (I-Bound) and (S-Bound) are satisfied at the admissible values of P_I and P_S. Henceforth, if this equilibrium exists, we will refer to it as the *insurance equilibrium*.

4.3.3 Pooling Equilibrium with Loan Sales

We continue by shifting our focus to the pooling equilibrium where both high and low types chose loan sales. We know $Prob(No\ Default|CRT) = 1 - Prob(Default|CRT) = 1 - \frac{p_l + qp_h}{q+1}$. The optimal prices ($P_I$,$P_S$) must satisfy:

$$P_S R_0 L_0 \geq (1 - p_l) R_0 L_0 \tag{S-PC1}$$

$$P_S R_0 L_0 + (1 - p_N) L_1 R_1 \geq R_0 L_0 (1 - p_h) + (1 - p_N) L_1 R_1 - p_N p_h B \tag{S-PC2}$$

$$P_S R_0 L_0 \geq (1 - p_l) R_0 L_0 - L P_I + p_l L \tag{S-IC1}$$

$$P_S R_0 L_0 + (1 - p_N) L_1 R_1 \geq (1 - p_h) R_0 L_0 - L_0 P_I + p_h L_0 + (1 - p_N) L_1 R_1 \tag{S-IC2}$$

$$1 - \frac{p_l + qp_h}{q+1} - P_S = 0 \tag{zero-π}$$

$$P_I \geq p_h \tag{I-Bound}$$

$$P_S \leq 1 - p_h \tag{S-Bound}$$

[17] Note that equilibrium refinements like the Cho-Kreps Intuitive Criterion have no bite in this setting (and in all equilibria to be shown in this paper). Therefore, we are able to focus on all off-the-equilibrium path beliefs that sustain the pooling equilibrium.

From (zero-π), we can see that $P_S = 1 - \frac{p_l + q p_h}{q+1}$. Given this, it is easy to verify that (S-PC1) is satisfied. As well, (S-PC2) is satisfied by assumption 1. Plugging the value for P_S into (S-IC1) yields:

$$P_I \geq p_l - R_0[\frac{q}{q+1}(p_l - p_h)] \tag{7}$$

Plugging P_S into (S-IC2) yields:

$$P_I \geq p_h + R_0[\frac{q}{q+1}(p_l - p_h)] \tag{8}$$

Since $R_0 > 1$, it follows that (8)$\Rightarrow$(7) and therefore, the insurance premium can take on any value in the range defined by the off-the-equilibrium path beliefs, (8). With the off-the-equilibrium path beliefs being defined by (8), we can assign an upper bound as: $P_I \leq p_l$. This restriction must hold because of the zero profit condition of the risk buyer.[18] Thus, $R_0 \leq \frac{q+1}{q}$ must hold for this pooling equilibrium to exist. We can see that if a project has a large return, then the bank will turn to the loan insurance market. It is easy to see that (I-Bound) and (S-Bound) are satisfied at the admissible values of P_I and P_S. Henceforth, if this equilibrium exists, we will refer to it as the *sales equilibrium.*

Duffee and Zhou (2001) conclude that if a sales market exists, and a loan insurance market is introduced, pooling in the sales market may no longer be possible. This in turn would cause the break down of the sales market altogether. Since Duffee and Zhou (2001) have insurance being written on a portion of the loan with no adverse selection, they show that the sales market can break down because of the flexibility of loan insurance. Our model produces a similar result without the flexibility in insurance, but from an entirely different channel. Since the sales market can exist in isolation with $R_0 > \frac{q+1}{q}$, if insurance is introduced under these circumstances, pooling in the sales market would not be possible. This would cause the sales market itself to break down.

All other equilibria can be ruled out in essentially the same way as was done above so the proofs are omitted.

We are now ready to analyze an enriched version of the model so that we can derive our main

[18] The reason for this is that if the bank is charged more than p_l for insurance, they would necessarily make positive profit since p_l is the highest probability the loan can default with.

results.

5 Moral Hazard, Relationship Management, and Costly Capital

One important point about having multiple equilibria is that there is no way of telling which will occur. This non-uniqueness stems from the fact that we have left key attributes of each instrument unmodelled thus far. Enhancing the model to a more realistic setup will give us insight into the choice of insurance versus sales. First, the relationship between a bank and a borrower can cause a moral hazard problem to develop. Consider a bank that has a special technology to verify that the firm is operating in a manner that is in keeping with the bank's interests. We refer to this as a monitoring technology. When the bank transfers away risk from a loan, it may no longer have the incentive to invest in this monitoring technology. In this paper, we do not analyze the origins of the moral hazard problem, but rather, we analyze the effect of its presence. For a review of moral hazard in banking, see for instance Gorton and Pennachhi (1995).

The second issue with CRT arises only with loan sales. In a loan sale, the underlying firm and new lender must expend resources to build a new relationship which can devalue the loan. We will refer to this cost as *relationship management*. In reality, the cost of selling a loan could go farther than just a devaluation of the current loan, as it could hurt future business with the underlying firm.[19] In some loan sale contracts, the underlying firm may even try to prevent a bank from selling their loan by specifying a no-sale stipulation. The costs associated with relationship management are not generally applicable to loan insurance because the originating bank maintains ownership of the loan and need not inform the underlying firm of their intent to insure. It has been shown empirically that these costs are present in loan sales. Dahiya et al. (2003) find that when a bank sells a loan, the market reacts negatively to it by devaluing the bank's stock.[20] With moral hazard and relationship management costs, conditions can be set so that the bank strictly prefers loan insurance in all states or vice versa. Moral hazard and relationship management will be analyzed in Section 5.1.

In Section 5.2, we relax the assumption that capital comes without cost. Realistically, banks

[19] We will not model this channel here.

[20] This evidence may also incorporate the adverse selection problem discussed before.

have investors who provide capital, but who expect a rate of return on their investment.[21] Competitiveness in the banking sector will provide us with different cost structures for sales and insurance. We will see that this addition has the possibility of making loan sales attractive to a bank who must acquire relatively costly capital.

The introduction of these two unique features are two of the contributions this paper makes to the literature. Duffee and Zhou (2001) does not consider the possible trade-offs between sales and insurance on these grounds. We will see that the addition of these two costs drives the equilibrium choice between sales and insurance, whereas Duffee and Zhou (2001) could make no such direct comparison.[22]

5.1 Moral Hazard and Relationship Management Costs

To consider the possibility that there are additional costs to using loan sales, we add an exogenous cost parameter $\alpha \in [0, 1]$. This new parameter represents the degree to which the project is worth less in the hands of the risk buyer due to both the moral hazard problem of the bank and the relationship management cost incurred by the risk buyer. When α is low, the costs associated with selling are high, and the bank must take a significantly lower price for the loan sale if it wishes to pursue this instrument of CRT. For simplicity, we assume that moral hazard and relationship effects are not present in loan insurance; however, the qualitative results will follow through if we allow for moral hazard in the loan insurance market. However, since the originating bank is still tied to the return of the loan, we would expect moral hazard to be smaller with loan insurance. This argument will be formalized below when we show how moral hazard can be endogenized in the model. If we consider a different setting where the bank maintains no ties to the return on the loan (i.e it insures both the principal and the return), then the moral hazard problem would be the same in insurance as in sales. However, α would still be larger for loan insurance because of the relationship management cost of loan sales. We can show the new expected profit from loan sales.

$$E(\pi_S) = \frac{1}{2}R_0L_0[1 - p_h(1 - q(1 - \alpha))] + \frac{1}{2}R_0L_0(1 - \alpha p_l) + q[(1 - p_N)L_1R_1]$$

[21] We will not analyze the choice between investor and depositor financing for the bank in this paper. We simply assume a rate of return that a bank must pay on any capital it uses.

[22] Recall that Duffee and Zhou (2001) differentiated sales and insurance through a maturity mismatch, which this paper contends is no longer a driving feature of these markets.

Not surprisingly, the expected profit from loan sales is unambiguously lower than that of loan insurance (as determined in (3)) when $\alpha < 1$. This result already gives us the intuition behind what will be the equilibrium outcome. Our main result, Proposition 3 will confirm that the smaller is α, the less likely it is that sales can be sustained in equilibrium.

It is important to recognize that this reduced form representation of moral hazard can be generated endogenously in an extension to the model. In Appendix B (section 7.2) we show that little is lost by assuming that moral hazard imposes an exogenous cost on the price of loan sales.

5.2 Costly Bank Capital

In this section, we generalize the previous exercises with the more realistic assumption that bank capital may come at a cost. An important question to ask is what would cause a bank to have different costs of capital? One key factor is how well capitalized a bank is. Investors in the bank will be willing to accept lower rates of return if the bank has sufficient equity to cover potential loss in case of default. If the bank is poorly capitalized, the risk to the investor is greater, and they will charge a greater amount representing the extra risk they must bear. However, we assume that the risk-buyer is not under this type of constraint.[23] Therefore, we assume that the risk buyer is not subjected to this cost of capital. This assumption can be relaxed so that the risk buyer does have a cost, but it is less than that of the bank.

Let there be two investors in the bank, an early investor, and a late investor. The early investor is endowed with unlimited capital at time $t = 0$, but none at times $t = 1, 2$. We represent their preferences as in Allen and Gale (2005) with the following risk-neutral utility function:

$$U(c_0, c_1, c_2) = (R_f - 1)c_0 + c_1 + c_2$$

where $R_f - 1$ represents the rental rate of capital and c_t is the consumption at time t. One of the key insights from the functional form is that investors are indifferent between consumption at $t = 1$ and $t = 2$. Because of this, they will require the same return, $R_f - 1 > 0$, regardless of how long they loan the capital to the bank. The late investor is endowed with unlimited capital at time

[23]This assumption is justified given our assumption that the risk buyer is well diversified.

$t = 1$, but none at time $t = 2$. We represent their utility function as:

$$U(c_0, c_1, c_2) = (R_f - 1)c_1 + c_2$$

This type of investor simply gives capital at time $t = 1$ and requires a return of $R_f - 1$ at time $t = 2$. Note that we have equalized the outside opportunity cost of each investor type for simplicity.[24] The rental rate of capital deserves some explanation. We use a rental rate to be consistent with the base case where the bank owns its own capital ($R_f = 1$). Alternatively, we could modify the base case so that the bank does not own its capital, but need not pay a return on it. The results do not change with either way of treating the capital cost. Therefore, we assume the bank will return the principal after the final date, but for simplicity, and without loss of generality, we normalize the principal to zero.

We now turn to the expected (ex-ante) profit for the bank. Recall we derived the expressions (3) and (4) earlier without costly capital. We can calculate the new expected costs of this additional feature. Denote the expected additional cost of insurance and sales with costly capital as $E(C_I)$ and $E(C_S)$ respectively.

$$\begin{aligned} E(C_I) &= \frac{1}{2}(1-q)(R_f-1)L_0 + q[(R_f-1)L_0 + (R_f-1)L_0P_I + (R_f-1)L_1] + \frac{1}{2}(1-q)(R_f-1)L_0 \\ &= (R_f-1)(L_0+qL_1) + (R_f-1)qL_0P_I \end{aligned}$$

$$\begin{aligned} E(C_S) &= \frac{1}{2}(1-q)(R_f-1)L_0 + q[(R_f-1)L_0 + (R_f-1)(L_1 - R_0L_0(P_S))] + \frac{1}{2}(1-q)(R_f-1)L_0 \\ &= (R_f-1)(L_0+qL_1) - (R_f-1)qR_0L_0P_S \end{aligned}$$

We can see that insurance is unambiguously more costly than sales. Therefore, loan sales yields more profit (ex-ante) than loan insurance under costly capital without moral hazard and relationship costs (i.e $\alpha = 1$). The intuition behind this result is that loan insurance forces the bank to obtain even more costly capital to engage in CRT ($(R_f - 1)qL_0P_I$). At the same time, loan sales allows them to free up capital (and reinvest the payoff from the sale) when they wish to pursue the new investment ($-(R_f - 1)qR_0L_0P_S$).

[24] We have also assumed that the two types of investors cannot trade with each other.

We will see that the intuition of the previous analysis is confirmed in Proposition 3, where we show that the higher is R_f, the less likely it is that insurance can be sustained in an equilibrium setting.

5.3 Moral Hazard, Relationship Management, and Costly Capital Together

A similar exercise to that of section 4.1 can give us the following assumption that permits the existence of our CRT markets in our generalized framework. The derivation can be found in Appendix C (section 7.3).

Assumption 2 $B \geq max\{\frac{L_0(R_f(P_I)-p_h)}{p_N p_h}, \frac{R_0 L_0((1-p_h)-\alpha P_S R_f)}{p_N p_h}\}$

We begin by ruling out all separating equilibria in this generalized setting.

Proposition 2 *No separating equilibria can exist in the generalized model where* $\alpha \in [0,1)$ *and* $R_f < 1$.

Proof. *See appendix.*

Let us now consider the insurance pooling equilibrium where banks with both high and low type loans choose to insure. To differentiate, given the knife-edge case where the incentive compatibility constraints hold with equality, we assume they choose insurance. This will make our incentive compatibility constraints in the sales equilibrium case strict. It can be shown that the incentive and participation constraints of the state {H,NEW} are implied by those in state {L,NEW} so we drop them. In what follows, we assume that the bank wishes to pursue full investment, as we did in the simpler case of the previous section. The optimal prices (P_I,P_S) are given by:

$$(1-p_l)R_0L_0 + p_lL_0 - (R_f-1)L_0 - R_fP_IL_0 \geq (1-p_l)R_0L_0 - (R_f-1)L_0 \tag{I-PC1}$$

$$\begin{aligned}(1-p_l)R_0L_0 + p_lL_0 + (1-p_N)L_1R_1 - (R_f-1)L_0 - R_fP_IL_0 - (R_f-1)L_1 \geq \\ (1-p_l)R_0L_0 + (1-p_N)L_1R_1 - (R_f-1)L_0 - (R_f-1)L_1 - p_Np_lB\end{aligned} \tag{I-PC2}$$

$$(1-p_l)R_0L_0 + p_lL_0 - (R_f-1)L_0 - R_fP_IL_0 \geq \alpha R_0L_0(P_S) - (R_f-1)L_0 \tag{I-IC1}$$

$$\begin{aligned}(1-p_l)R_0L_0 + p_lL_0 + (1-p_N)L_1R_1 - (R_f-1)L_0 - R_fP_IL_0 - (R_f-1)L_1 \geq \\ \alpha R_0L_0(P_S) + (1-p_N)L_1R_1 - (R_f-1)L_0 - (R_f-1)[L_1 - (\alpha RP_SL_0)]\end{aligned} \tag{I-IC2}$$

$$L_0(P_I - \frac{p_l + qp_h}{q+1}) = 0 \tag{zero-π}$$

$$P_I \geq \frac{p_h}{R_f} \tag{I-Bound}$$

$$\alpha P_S \leq 1 - p_h \tag{S-Bound}$$

On the left hand side of (I-IC1) and (I-IC2), we see that with insurance, the bank holds the investment for two periods, and incurs a cost of $(R_f - 1)L$. As well, they borrow an additional L_0P_I for one period to pay for the cost of insuring, and incur a cost of $R_fL_0P_I$.

On the right hand side of (I-IC1) and (I-IC2), the cost of capital for loan sales deserves some explanation. The bank acquires the capital for the initial loan at a cost of $(R_f - 1)L$. At time $t = 1$, they need not borrow the full amount of capital for the new investment. This is because they can reinvest the proceeds of the loan sale. The cost of the extra capital that is needed for the new investment is $(R_f - 1)[L_1 - \alpha R_0P_SL_0]$. For simplicity we assume that $L_1 \geq \alpha R_0P_SL_0$.[25] This assumption is innocuous since we will soon see that our characterizing solutions do not depend on L_1.

Given (zero-π), we know that $P_I = \frac{p_l+qp_h}{q+1}$. (I-PC1) is satisfied when $R_f \leq \frac{p_l}{P_I}$, while (I-PC2) is implied by assumption 2. P_S is given by the off-the-equilibrium path beliefs of the risk buyer and

[25] This assumption is equivalent to the assumption that the bank has a storage technology with a return of 1.

can be defined given α and R_f. We obtain the following parameterizations for (I-IC1) and (I-IC2):

$$\alpha \leq \frac{R_0(1-p_l) - R_f P_I + p_l}{R_0 P_S} \tag{I-IC1a}$$

$$\alpha \leq \frac{R_0(1-p_l) - R_f P_I + p_l}{R_0 P_S R_f} \tag{I-IC2a}$$

Since (I-IC1a) and (I-IC2a) differ by a fraction $\frac{1}{R_F}$, it follows that (I-IC2) $\Rightarrow$ (I-IC1). Therefore, (I-IC2) is binding, while (I-IC1) is slack. We can use a standard approach to find out when this equilibrium cannot exist. We let the off-the-equilibrium path beliefs be $P_S = 1 - p_l$. Therefore, if the equilibrium cannot exist under this condition, it cannot exist for any valid off-the-equilibrium path belief. Substituting $P_S = 1 - p_l$ into (I-IC2) yields this range:

$$\alpha > \frac{R_0(1-p_l) - R_f P_I + p_l}{R_0(1-p_l) R_f}$$

We continue by analyzing when the equilibrium can exist.

Lemma 3 *The insurance equilibrium exists whenever one of the following two conditions is met*

1. $\alpha < \frac{P_I(1-p_l)}{P_S p_l}$ *and* $1 \leq R_f \leq \frac{p_l}{P_I}$
2. $\alpha > \frac{P_I(1-p_l)}{P_S p_l}$ *and* $1 \leq R_f \leq \frac{R_0(1-p_l)+p_l}{\alpha R_0 P_S + P_I}$

Proof. *See appendix.*

The results of this Lemma are relatively straight-forward. The first condition says that if α is small, then the insurance equilibrium will exist when R_f (low cost of capital) is sufficiently small. The second condition says if α is larger, we will require an even smaller value of R_f than what was required in the first condition.

We can conduct a similar exercise for the sales equilibrium. The participation and incentive constraints of state {L,NEW} are implied by those of state {H,NEW} and are dropped. The following setup will yield the optimal prices (P_I,P_S):

$$\alpha P_S R_0 L_0 - (R_f - 1)L_0 \geq (1 - p_l)R_0 L_0 - (R_f - 1)L_0 \tag{S-PC1}$$

$$\alpha P_S R_0 L_0 - (R_f - 1)L_0 + (1 - p_N)L_1 R_1 - (R_f - 1)[L_1 - (\alpha R P_S L_0)] \geq \tag{S-PC2}$$

$$(1 - p_h)R_0 L_0 + (1 - p_N)L_1 R_1 - (R_f - 1)L_0 - (R_f - 1)L_1 - p_N p_h B$$

$$\alpha P_S R_0 L_0 - (R_f - 1)L_0 > (1 - p_l)R_0 L_0 + p_l L_0 - (R_f - 1)L_0 - R_f P_I L_0 \tag{S-IC1}$$

$$\alpha P_S R_0 L_0 - (R_f - 1)L_0 + (1 - p_N)L_1 R_1 - (R_f - 1)[L_1 - (\alpha R P_S L_0)] > \tag{S-IC2}$$

$$(1 - p_h)R_0 L_0 + p_h L_0 + (1 - p_N)L_1 R_1 - (R_f - 1)L_0 - R_f P_I L_0 - (R_f - 1)L_1$$

$$R_0 L_0[\alpha(1 - \frac{p_l + q p_h}{q + 1}) - \alpha P_S] = 0 \tag{zero-π}$$

$$P_I \geq \frac{p_h}{R_f} \tag{I-Bound}$$

$$\alpha P_S \leq 1 - p_h \tag{S-Bound}$$

From (zero-π) we know that $P_S = 1 - \frac{p_l + q p_h}{q+1}$. (S-PC1) holds when $\alpha \geq \frac{1-p_l}{P_S}$, while (S-PC2) holds by assumption 2. P_I is given by the off-the-equilibrium path beliefs of the risk buyer and can be defined given α and R_f. We can find a parametrization in terms of α for (S-IC1) and (S-IC2):

$$\alpha > \frac{R_0(1 - p_l) + p_l - R_f P_I}{R_0 P_S} \tag{S-IC1b}$$

$$\alpha > \frac{R_0(1 - p_h) + p_h - R_f P_I}{R_f R_0 P_S} \tag{S-IC2b}$$

Using the same method as in the insurance case, we can determine when the sales equilibrium cannot exist. By substituting $P_I = p_l$ as the off-the-equilibrium path belief into (S-IC1b) and (S-IC2b), we can obtain the range for which sales cannot exist (if either one of the following two conditions are met):

$$\alpha \leq \frac{R_0(1 - p_l) + p_l - R_f p_l}{R_0 P_S} \tag{9}$$

$$\alpha \leq \frac{R_0(1 - p_h) + p_h - R_f p_l}{R_f R_0 P_S} \tag{10}$$

The following Lemma gives the formal conditions for when the sales equilibrium exists.

Lemma 4 *The sales equilibrium exists whenever one of the following three conditions is met*

1. $\alpha \geq \frac{1-p_l}{P_S}$ *and* $R_f > \max\{\frac{R_0(1-p_h)+p_h}{(1-p_l)R_0+P_I}, \frac{p_l}{P_I}\}$
2. $\frac{R_0(1-p_h)+p_h}{P_I+R_0P_S} < R_f \leq \frac{R_0(1-p_h)+p_h}{(1-p_l)R_0+P_I}$ *and* $\alpha > \frac{R_0(1-p_h)+p_h-R_fP_I}{R_fR_0P_S} \geq \frac{R_0(1-p_l)+p_l-R_fP_I}{R_0P_S}$
3. $\frac{R_0(1-p_l)+p_l-R_0P_S}{P_I} < R_f \leq \frac{p_l}{P_I}$ *and* $\alpha > \frac{R_0(1-p_l)+p_l-R_fP_I}{R_0P_S} \geq \frac{R_0(1-p_h)+p_h-R_fP_I}{R_fR_0P_S}$

Proof. *See appendix.*

The first condition says that so long as α and R_f are sufficiently high, then this equilibrium can exist. The second two conditions simply say that if we force α to be even higher than the first condition, then we can sustain this equilibrium for lower costs of capital (smaller values of R_f).[26] When we combine this Lemma with that of Lemma 3, Proposition 3 will show that the bank will tend to rely on loan insurance when α is low or R_f is close to one. For example, it could be that banks are well capitalized and/or the moral hazard or relationship banking concerns are troublesome in the loan sales market. When α is low and R_f is high, both markets may not exist. By assumption 2, we can rule out the case where a bank with one type of loan wishes not to participate. We can fix this idea by defining a particular off-the-equilibrium path belief in each of these two equilibria. In the insurance case, we set $P_S = 1 - \frac{p_l+qp_h}{q+1}$, and for the sales case we set $P_I = \frac{p_l+qp_h}{q+1}$. The following proposition shows under this off-the-equilibrium path belief, the choice between insurance and sales is unique.

Lemma 5 *Under the off-the-equilibrium path beliefs assigned, the equilibrium is uniquely determined by R_f and α as either insurance, sales or neither.*

Proof. *See appendix.*

We can now give our main result of the section. The following Proposition states that when capital is relatively cheap, then the insurance equilibrium can be supported for when there is sufficient moral hazard/relationship management costs. Conversely, when the moral hazard/relationship

[26]Note that in the second two conditions, one must be careful as the the lower bound on R_f cannot be smaller than 1.

management costs are low, the sales equilibrium can be supported for sufficiently high costs of capital. The proof of this Proposition follows easily from Lemmas 3 and 4. We can obtain uniqueness of the equilibrium from Lemma 5.

Proposition 3

1. *When the cost of capital is low, the bank will use insurance when α is sufficiently small.*

2. *When the costs of moral hazard/relationship management are low, the bank will use sales when R_f is sufficiently high.*

The reason for this result has been discuss earlier, but will be reiterated for clarity. The bank may choose sales over insurance when the cost of capital is high because insurance requires an upfront payment, whereas sales frees up capital immediately. Conversely, since the moral hazard and relationship management problems will tend to be worse for loan sales, the bank will use insurance when these costs are high.

The results of this paper show that by introducing adverse selection into the insurance market, the Duffee and Zhou (2001) framework changes quite a bit. In Duffee and Zhou (2001), the existence of the sales equilibrium versus the insurance equilibrium was driven by the timing of the model, and the specific informational assumption on loan insurance. In this model, by relaxing these two key assumptions (which we discuss why they may be unrealistic), we derive properties of a bank (or loan) which can determine whether sales or insurance would be used.

6 Conclusion

We use a model where CRT arises because of two factor: first, a bank can use CRT to dump low quality loans, and second, a bank can use CRT when its total risk exceeds a pre-determined level. We show that in the basic setup with no moral hazard or relationship management costs, only an insurance or sales pooling equilibrium can exist. To determine the conditions under which either equilibrium can be the unique outcome, we extend the model to allow for costly capital, moral hazard and relationship banking issues. We find that well capitalized banks will use loan insurance in the presence of moral hazard and relationship costs of loan sales. Finally, we show that if the bank is poorly capitalized, so that capital is very costly, they may be forced into the loan sales market even in some cases where the loan sale price could be significantly depressed.

7 Appendix

7.1 Appendix A

Proof of Lemma 1

Conditional on the bank pursuing full initial investment, we can consider the two states in which the bank may want to avoid investment in the new project separately: {H,NEW} and {L,NEW}. We begin by looking at {H,NEW} and finding the range of B where the bank will wish to pursue the new investment:

$$\begin{aligned} R_0 L_0(1-p_h) + R_A L_2(1-p_N) - p_N p_h B &\geq R_0 L_0(1-p_h) \\ \Rightarrow B &\leq \frac{R_A L_2(1-p_N)}{p_N p_h} \end{aligned} \tag{11}$$

We now derive the condition for full new investment in the state {L,NEW}:

$$\begin{aligned} R_0 L_0(1-p_l) + R_A L_2(1-p_N) - p_N p_l B &\geq R_0 L_0(1-p_l) \\ \Rightarrow B &\leq \frac{R_A L_2(1-p_N)}{p_N p_l} \end{aligned} \tag{12}$$

Because $p_l > p_h$ (12) $\Rightarrow$ (11), and thus (12) is the only parametrization needed to ensure the new investment is pursued when it is available.

If the bank does not pursue full initial investment, then it must always be optimal to pursue the new investment since it has positive expected return, and the possibility of the loss B has already been eliminated. This concludes the proof. $\blacksquare$

Proof of Lemma 2

For the first part of the proposition, consider investing $L \in (\hat{L} - L_1, L_0)$. This investment is strictly dominated by $L = L_0$ since the project is of positive net present value (NPV), and by investing $L \in (\hat{L} - L_1, L_0)$, you are still subjected to the possibility of the loss, B. Next, consider the case in which $L < \hat{L} - L_1$. This investment level is strictly dominated by $L = \hat{L} - L_1$ since the project is of positive NPV, and by choosing $L = \hat{L} - L_1$, the possibility of the loss of B is still eliminated. $L > L_0$ is not possible since the firm does not request more than L_0 units from the bank.

From the first part of the proposition, we need only focus on two potential levels of investment to address the second part: $L = \hat{L} - L_1$ and $L = L_0$. First, consider the case in which the bank invests $L = \hat{L} - L_1$ and assume that $B \leq \frac{R_A L_2 (1-p_N)}{p_N p_l}$:

$$E(\pi_{L=\hat{L}-L_1}) = \frac{1}{2}(1-p_h)R_0(\hat{L}-L_1) + \frac{1}{2}(1-p_l)R_0(\hat{L}-L_1) + q(1-p_N)L_1 R_1 \tag{13}$$

We now consider the case in which the bank invests the requested $L_0 = L$:

$$E(\pi_{L=L_0}) = \frac{1}{2}(1-p_h)R_0 L_0 + \frac{1}{2}(1-p_l)R_0 L_0 + q(1-p_N)L_1 R_1 - \frac{1}{2}q(p_h+p_l)p_N B \tag{14}$$

Comparing (13) and (14) we derive the condition in which full initial investment takes place:

$$L_0 - (\hat{L} - L_1) \geq \frac{Bqp_N(p_h + p_l)}{R_0[(1-p_h) + (1-p_l)]} \tag{15}$$

Note that if $L_0 - (\hat{L} - L_1) < \frac{Bqp_N(p_h+p_l)}{R_0[(1-p_h)+(1-p_l)]}$, then the bank under-invests in the initial loan and pursues the new loan. Next, consider the case where $B > \frac{R_A L_2(1-p_N)}{p_N p_h}$. In this case, the bank can either have full investment in the initial loan and does not pursue the new loan, or can under-invest in the initial loan, and fully pursue the new loan. The payoff to under-investing is given in (13) while the payoff to full investment are given is:

$$E(\pi_{L=L_0}) = \frac{1}{2}(1-p_h)R_0 L_0 + \frac{1}{2}(1-p_l)R_0 L_0 \tag{16}$$

Comparing (13) and (16), we obtain the following condition for full investment:

$$L_0 - (\hat{L} - L_1) \geq \frac{2q(1-p_N)L_1 R_1}{R_0[(1-p_h) + (1-p_l)]}$$

The final case is where $B \in (\frac{R_A L_2(1-p_N)}{p_N p_l}, \frac{R_A L_2(1-p_N)}{p_N p_l}]$. If the banks invests fully at time $t = 0$, then, conditional on the new investment being available, the bank will invest in it only if it is revealed that the initial loan is of high quality. The payoff to under-investment is given is (13), while the payoff to full investment is given by:

$$E(\pi_{L=L_0}) = \frac{1}{2}(1-p_h)R_0L_0 + \frac{1}{2}(1-p_l)R_0L_0 + \frac{1}{2}(q(1-p_N)L_1R_1 - Bp_Np_l) \tag{17}$$

Comparing (13) and (17), we obtain the following condition for full investment:

$$L_0 - (\hat{L} - L_1) \geq \frac{q(1-p_N)L_1R_1 - Bp_Np_l}{R_0[(1-p_h)+(1-p_l)]}$$

■

Proof of Proposition 1

Comparing (13) and (3), we can see that the existence of either one of the CRT instruments solves the under-investment problem that can occur if (15) is not satisfied.

We give the expected, ex-ante profits of a bank that does not pursue the new investment in {H,NEW} (denoted by NEW_H), {L,NEW} (denoted by NEW_L) or both (denoted by NONEW).

$$E(\pi^{NONEW}_{L_0=L}) = \frac{1}{2}(1-p_h)R_0L_0 + \frac{1}{2}(1-p_l)R_0L_0 \tag{18}$$

$$E(\pi^{NEW_H}_{L_0=L}) = \frac{1}{2}(1-p_h)R_0L_0 + \frac{1}{2}(1-p_l)R_0L_0 + \frac{1}{2}q(1-p_N)L_1R_1 - \frac{1}{2}qp_Np_hB \tag{19}$$

$$E(\pi^{NEW_L}_{L_0=L}) = \frac{1}{2}(1-p_h)R_0L_0 + \frac{1}{2}(1-p_l)R_0L_0 + \frac{1}{2}q(1-p_N)L_1R_1 - \frac{1}{2}qp_Np_lB \tag{20}$$

Comparing (18), (19) and (20) with (3), we see that CRT also ensures that the new investment will be fully pursued. Since this is not the case without CRT, and these projects are of positive net present value, we conclude that CRT induces the optimal investment level. This is because under-investment involves leaving a portion of the project unfunded, and forcing it to proceed on a smaller scale. Comparing the expected profit from full investment under CRT and no CRT, we immediately see that the use of CRT is always more profitable for the bank. ■

Proof of Proposition 2

First, consider the separating equilibrium where the high types chose sales, and the low types chose insurance. The zero profit condition tells us that $P_S = 1 - p_h$ and $P_I = p_l$. The participation constraint in the state $\{L, NONEW\}$: (PC1) can be written as:

$$(1-p_l)R_0L_0 - R_fp_lL_0 + p_lL_0 - (R_f-1)L_0 \geq (1-p_l)R_0L_0 - (R_f-1)L_0$$
$$\Rightarrow R_f \leq 1$$

Since $R_f > 1$, (PC1) will never be satisfied.

Next, consider the separating equilibrium where the high types chose to insure, and the low types chose to sell. The zero profit condition tells us that $P_S = 1-p_l$ and $P_I = p_h$. The participation constraint in the state $\{L, NONEW\}$ (PC1) can be written as:

$$\alpha(1-p_l)R_0L_0 - (R_f-1)L \geq (1-p_l)R_0L_0 - (R_f-1)L_0$$
$$\Rightarrow \alpha \geq 1$$

Since $\alpha < 1$, (PC1) will never be satisfied.

Any equilibria where either type is indifferent between loan sales and loan insurance will yield either one of the two previous cases and can be ruled out. ■

Proof of Lemma 3

There are two cases that we need to consider since the binding constraint will depend on the parameters of the model.

The first condition is derived assuming that (I-PC1) is the binding constraint. We then put the necessary restriction on (I-IC2) to make (I-PC1) bind.

$$\frac{R_0(1-p_l)+p_l}{\alpha R_0P_S + P_I} > \frac{p_l}{P_I}$$
$$\Rightarrow \alpha < \frac{P_I(1-p_l)}{P_Sp_l}$$

The second condition assume (I-IC2) binds. We put the necessary restriction on R_f from (I-PC1) to make sure this is the case.

$$\frac{p_l}{P_I} > \frac{R_0(1-p_l)+p_l}{\alpha R_0 P_S + P_I}$$
$$\Rightarrow \alpha > \frac{P_I(1-p_l)}{P_S p_l}$$

■

Proof of Lemma 4

There are three cases that we need to consider since the binding constraint will depend on the parameters of the model.

The first condition is derived assuming that (S-PC1) is the binding constraint. We find the range of R_f such that the R.H.S of (S-IC1b) and (S-IC2b) are less than $\frac{1-p_l}{P_S}$.

$$\frac{R_0(1-p_l)+p_l-R_f P_I}{R_0 P_S} < \frac{1-p_l}{P_S}$$
$$\Rightarrow R_f > \frac{p_l}{P_I}$$

$$\frac{R_0(1-p_h)+p_h-R_f P_I}{R_f R_0 P_S} < \frac{1-p_l}{P_S}$$
$$\Rightarrow R_f > \frac{R_0(1-p_h)+p_h}{(1-p_l)R_0+P_I}$$

The second condition assumes that (S-IC2b) binds. The condition on R_f allows the R.H.S of (S-IC2b) to be less than $\frac{1-p_l}{P_S}$. The second condition results because for (S-IC2b) to bind, the R.H.S of (S-IC2b) must be greater than the R.H.S of (S-IC1b).

$$\frac{R_0(1-p_h)+p_h-R_f P_I}{R_f R_0 P_S} \geq \frac{1-p_l}{P_S}$$
$$\Rightarrow R_f \leq \frac{R_0(1-p_h)+p_h}{(1-p_l)R_0+P_I}$$

To obtain a lower bound on R_f, we need to make sure that the value or R_f is not so low as to require $\alpha > 1$. To this we compute:

$$\frac{R_0(1-p_h)+p_h-R_fP_I}{R_fR_0P_S} \leq 1$$
$$\Rightarrow R_f \geq \frac{R_0(1-p_h)+p_h}{P_I+R_0P_S}.$$

The third condition assumes that (S-IC1b) binds. The condition on R_f allows the R.H.S of (S-IC1b) to be less than $\frac{1-p_l}{P_S}$. The second condition results because for (S-IC1b) to bind, the R.H.S of (S-IC1b) must be greater than the R.H.S of (S-IC2b).

$$\frac{R_0(1-p_l)+p_l-R_fP_I}{R_0P_S} \geq \frac{1-p_l}{P_S}$$
$$\Rightarrow R_f \leq \frac{p_l}{P_I}$$

To obtain a lower bound on R_f, we need to make sure that the value or R_f is not so low as to require $\alpha > 1$. To this we compute:

$$\frac{R_0(1-p_l)+p_l-R_fP_I}{R_0P_S} \leq 1$$
$$\Rightarrow R_f \leq \frac{R_0(1-p_l)+p_l-R_0P_S}{P_I}.$$

■

Proof of Lemma 5

Plugging in $P_S = 1 - P_I$ into (I-IC1a), (S-IC1b) and (S-IC2b). If (S-IC1b) is the binding constraint for the sales equilibrium, then the set $S\left(\alpha|\text{(I-IC1a)} \cap \text{(S-IC1b)}\right)$ is empty. This implies that sales and insurance are mutually exclusive. Furthermore, we can see that in this case, either of the two cases must occur. If (S-IC2b) is the binding constraint for the sales equilibrium, so that $\frac{R_0(1-p_l)+p_l-R_fP_I}{R_0P_S} < \frac{R_0(1-p_h)+p_h-R_fP_I}{R_fR_0P_S}$, then the set $S\left(\alpha\middle|\frac{R_0(1-p_l)-R_fP_I+p_l}{R_0P_S} < \alpha \leq \frac{R_0(1-p_h)+p_h-R_fP_I}{R_fR_0P_S}\right)$ is non-empty so that neither the insurance nor sales equilibrium exists. Furthermore, it is easy to see that the insurance and sales equilibrium cannot co-exist in this case. ■

7.2 Appendix B

Consider a bank with access to an unverifiable monitoring technology at time $t = 1$ that has a cost, e. Let us assume that without this monitoring, all low quality loans will fail with probability 1.[27] Consider the case in which there is no CRT available. To ensure that the bank wishes to monitor, the following condition must hold:

$$e \leq R_0 L_0 (1 - p_l)$$

Next, consider the case of loan insurance. There is a trade-off present with this new monitoring technology. The bank can choose not to monitor, but give up the potential return from the low quality loans.[28] We can put the following assumption on e to ensure that they wish to continue monitoring in the low state when they insure their loan.

$$\begin{aligned} R_0 L_0 (1 - p_l) + p_l L_0 - e &\geq L_0 \\ \Rightarrow e &\leq (R_0 - 1)(1 - p_l) \end{aligned}$$

Finally, if the bank wishes to use loan sales, it can never credibly commit to monitoring the bad loans for any $e > 0$. Therefore, the price of the loan sale will simply be $R_0 L_0 [1 - Prob(Default|sales)] = R_0 L_0 [1 - \frac{1+qp_h}{q+1}]$. We can see immediately that this new loan sales price is smaller than the original price without moral hazard. We therefore use the exogenous variable α to represent the amount that the loan sale price is reduced with moral hazard present.[29] Intuitively, if $\alpha < 1$, all else equal, the bank may not wish to use loan sales and the market may not exist. For example, consider the participation constrain in the state {L,NONEW} of the sales equilibrium:

$$\alpha P_S R_0 L_0 \geq (1 - p_l) R_0 L_0$$

[27] The qualitative results follow through if we make the assumption that bank monitoring can transform low quality loans into high quality loans.

[28] If the bank chooses to not monitor the bad loans we know that $Prob(Default) = \frac{1+qp_h}{q+1}$.

[29] Gorton and Pennachhi (1995) and DeMarzo and Duffie (1999) show that if the bank retains a portion of the loan (usually first-loss), the moral hazard can be lowered. A modern example is that of a Collateralized Loan Obligation (CLO). We will not consider tranching in this paper.

It follows that if $\alpha < \frac{1-p_l}{P_S}$, the participation constraint can never be satisfied, and therefore they will not sell their loan in this state.

7.3 Appendix C

We now consider the resulting equilibrium when moral hazard, relationship management and costly capital are added to the analysis. We begin the analysis by redefining the parameter space of interest. We turn to the state {H,NONEW} first.

$$\pi_{NI} = R_0(1-p_h)L_0 - (R_f - 1)L_0$$
$$\pi_I = R_0(1-p_h)L_0 + p_h L_0 - (R_f - 1)L_0 - R_f L_0 P_I$$

It follows that for $\pi_{NI} \geq \pi_I$, the condition $P_I \geq \frac{p_h}{R_f}$ must be added to the optimal contracting problem as (I-Bound). We now analyze {H,NEW} to see under what condition they will use loan insurance.

$$\begin{aligned}
\pi_{NI} &= R_0(1-p_h)L_0 - (R_f-1)L_0 + (1-p_N)L_1R_1 - p_N p_h B - (R_f-1)L_1 \\
\pi_I &= R(1-p_h)L_0 + p_h L_0 + (1-p_N)L_1R_1 - (R_f-1)L_0 - R_f P_I L_0 - (R_f-1)L_1
\end{aligned}$$

For $\pi_I \geq \pi_{NI}$ the following must hold:

$$B \geq \frac{L_0(R_f P_I - p_h)}{p_N p_h} \tag{21}$$

Therefore, (21) gives us the parameter bound on B. To find a similar bound for loan sales, we begin by looking at {H,NONEW}.

$$\pi_{NS} = R_0(1-p_h)L_0 - (R_f - 1)L_0$$
$$\pi_S = \alpha R_0(P_S)L_0 - (R_f - 1)L_0$$

For $\pi_{NS} \geq \pi_S$, we will add $\alpha P_S \leq 1 - p_h$ to our optimal contracting problem as (S-Bound). We now analyze {H,NEW} to see under what condition they will use loan sales:

$$\begin{aligned}
\pi_{NS} &= R_0 L_0 (1-p_h) - (R_f - 1)L_0 + (1-p_N)L_1 R_1 - (R_f - 1)L_1 - p_h p_N B \\
\pi_S &= \alpha R_0 L_0 (P_S) + (1-p_N)L_1 R_1 - (R_f - 1)L_0 - (R_f - 1)[L_1 - \alpha R_0 P_S L_0]
\end{aligned}$$

From above, for $\pi_S \geq \pi_{NS}$, the following must hold:

$$B \geq \frac{R_0 L_0 ((1-p_h) - \alpha P_S R_f)}{p_N p_h} \tag{22}$$

The parametrization that characterizes loan sales is given by (22).

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Figure 1: Growth in Credit Derivatives

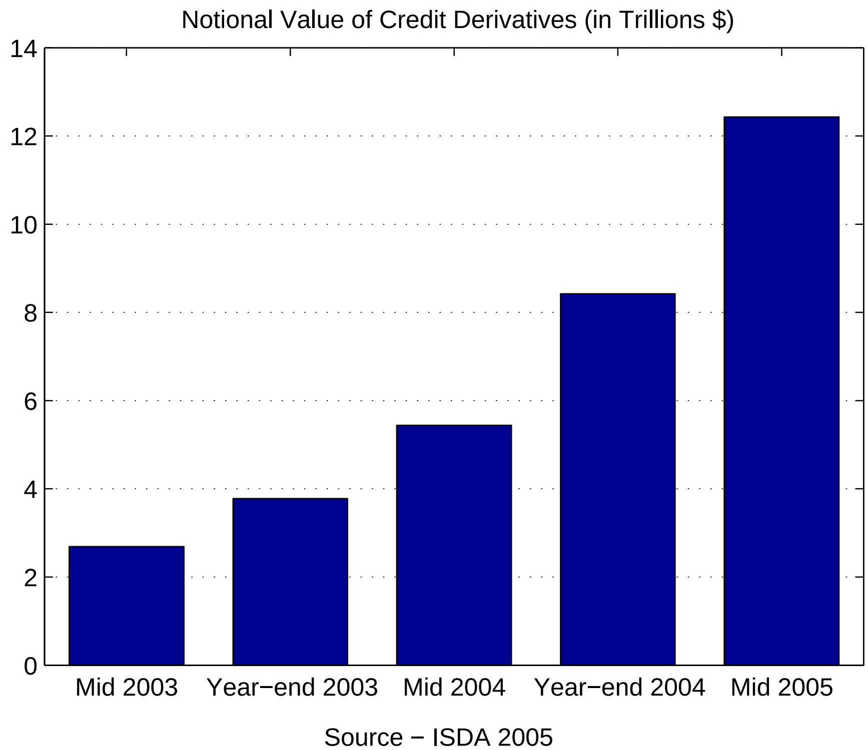

Figure 2: Average weekly trading volume (per institution) of three common types of derivatives

Instrument	2003	2004	2005
Currency Options	427	559	905
Equity Derivatives - Vanilla	291	153	153
Credit Derivatives	**79**	**103**	**206**

Source - ISDA 2005 Operations Benchmark Survey