



Queen's Economics Department Working Paper No. 1090

# Growth, Cycles and Welfare: A Schumpeterian Perspective

Patrick Francois  
University of British Columbia

Huw Lloyd-Ellis  
Queen's University

Department of Economics  
Queen's University  
94 University Avenue  
Kingston, Ontario, Canada  
K7L 3N6

9-2006

# Growth, Cycles and Welfare: A Schumpeterian Perspective\*

Patrick Francois  
Department of Economics  
University of British Columbia  
Vancouver, B.C., Canada.  
CentER, Tilburg University  
The Netherlands  
CEPR and CIAR  
francois@interchange.ubc.ca

Huw Lloyd–Ellis  
Department of Economics  
Queen’s University  
Kingston, Ontario, Canada  
and  
CIRPÉE  
lloydell@qed.econ.queensu.ca

September 2006

## Abstract

We use a Schumpeterian model in which both the economy’s growth rate and its volatility are endogenously determined to assess some welfare and policy implications associated with business cycle fluctuations. Because it features a higher average growth rate than its acyclical counterpart, steady–state welfare is higher along the cyclical equilibrium growth path of the model. We assess the impact of alternative stabilization policies designed to smooth cyclical fluctuations. Although, it is possible to significantly reduce the variance of output growth via simple policy measures, the welfare benefits are at best negligible and at worst completely offset by the resulting reduction long–term productivity growth.

---

\*This is a modified and extended version of the paper entitled “Schumpeterian Restructuring” presented at the CREI–World Bank Conference on the Growth and Welfare Effects of Macroeconomic Volatility in Barcelona February 2006. We thank Thierry Tresselt for a helpful discussion.

# 1 Introduction

A common presumption amongst economists and policy-makers is that business cycle fluctuations impose significant welfare costs. One potential source of such costs is the volatility of consumption faced by risk-averse households. However, as Lucas (1987) famously demonstrated, in the context of a representative household model with CES preferences, the volatility of US aggregate consumption is too low to impose much of a welfare cost. An alternative potential source of welfare costs is emphasized by Barlevy (2004a) in an AK model with adjustment costs — variability in investment levels leads to less productive investment on average and, hence, to an endogenously lower long-run growth rate. In this article, we consider the welfare implications of fluctuations and growth in the context of an economy in which cyclical fluctuation, as well as growth, are endogenously determined. We abstract from the impacts of uncertainty, and focus instead on the role played by trade-offs between innovative activity and fixed capital formation over the business cycle. In contrast to the standard presumption, we show that cycles can raise welfare by inducing greater innovation on average and that stabilization policies which reduce volatility can often be welfare reducing because of their detrimental impact on long-term productivity growth.

Lucas (1987) compared utility along a smoothly growing consumption path with that along a cyclical one. Both cyclical variation and trend growth were exogenously given and there was no underlying model of the macroeconomy. In this simple framework he asked what percentage of yearly consumption a representative household would be willing to forego to be free of the variations in consumption. The answer he found was not much (0.1 of a percent). In contrast he found that a household would be willing to give up a large proportion (20%) of yearly consumption to increase the growth rate by a mere one per cent, leading him to conclude that, the welfare consequences of business cycle fluctuations pale in comparison to those of growth. Subsequent papers have explored extensions of Lucas' parsimonious set-up. Imrohoglu (1999) and Atkeson and Phelan (1995) relax the assumption of complete markets, but still report small gains to eradicating fluctuations. A number of others — Obstfeld (1994), Pemberton (1996) and Dolmas (1998) — explore the implications of a non-expected utility representation but also find relatively small gains. Larger gains arise when habit formation is included in preferences (see Van Wincoop (1994), Campbell (1998) and Campbell and Cochrane (1999)), and if various kinds of agent heterogeneity are allowed (see Beaudry and Pages, 1996, Gomes Greenwood and Rebello, 1998, and Storesletten, Telmer and Yaron 2000).<sup>1</sup>

Barlevy (2004a) considers a model in which fluctuations are driven by exogenous productivity shocks, but long-run growth is endogenously determined and is affected by the variance of these shocks. The connection between fluctuations and growth that he explores arises if the growth rate is concave in the rate of investment. Because investment alternates between high levels during expansions when marginal costs are high, and low levels during contractions when marginal costs are low, there is an efficiency and growth

---

<sup>1</sup>For a comprehensive survey of this literature see Barlevy (2004b).

cost associated with productivity shocks. He estimates a potentially large welfare improvement resulting from the effect on investment and hence growth of removing the shocks. A key insight of Barlevy's work is that a possible avenue by which welfare may be adversely affected is when cycles affect the economy's growth rate.

In this article, we study an economy in which, not just the growth rate, but the economy's cyclical behaviour itself is an equilibrium outcome. In Francois and Lloyd-Ellis (2003) we develop a basic Schumpeterian paradigm in which business cycle fluctuations are an intrinsic part of the growth process — expansions reflect the endogenous, clustered implementation of productivity improvements, and recessions are the negative side-product of the restructuring that anticipates them. In Francois and Lloyd-Ellis (2006) we extend the model to allow for (potentially reversible) capital accumulation and show how fluctuations in the investment rate support the incentives needed to generate the multi-sector cycle. This extension is particularly important for an analysis of the welfare implications of cycles because it implies that consumption is partially smoothed relative to output. The welfare implications of cycles in an economy without such smoothing overstates the volatility of consumption and hence the costs of business cycles.

Our analytical framework has two key implications which we emphasize:

- Eradicating cycles is not just a question of shutting down some exogenous productivity process, but involves switching to a distinct equilibrium growth path. The economy has both an acyclical and a cyclical equilibrium growth path. Although the cyclical growth path exhibits greater consumption volatility than the corresponding acyclical one, it also generates higher growth. We find that the growth effect on welfare dominates the volatility effect — in our calibrated simulation exercise, for example, welfare is over 30% lower along the acyclical path. To understand why, note that along the acyclical growth path, productivity improvements are implemented immediately. Consequently the opportunity cost (due to lost production) of searching for new ideas rises in proportion to the growth in their value, with no reallocation of labor effort across sectors. Along the cyclical growth path implementation is delayed during downturns, so that the opportunity cost would can only rise in proportion to their value if search intensity accelerates. During expansions the opportunity cost of search rises so much relative to the benefits that to equate them would require more labor effort to be allocated into production than is available. Since search effort is bounded below by zero, it follows that the average level of entrepreneurship, and hence growth, is higher in the cyclical equilibrium.

- In contrast to other studies, it is possible to consider the impact of policy measures designed to “stabilize” the economy by reducing the variance of output growth. We consider 3 simple policy measures: taxes on profits, state-dependent taxes or subsidies to capital accumulation, and Keynesian-style counter-cyclical tax/transfer policies. We show that all three of these policy measures can be designed to substantially reduce the volatility of output growth, thereby stabilizing the economy. However, in steady-state, any

welfare gains resulting from this stabilization are minute because the impacts on consumption volatility are so small. Moreover, while capital taxes and counter-cyclical tax/transfers can be designed in a way that is neutral with respect to long run growth, increased profit taxes stifle growth by mitigating the incentives to innovate and thereby reduce welfare. Of the set of policies that we analyze, none are able to simultaneously raise growth and reduce fluctuations.

The paper proceeds as follows. In Section 2 we set up the model and develop an explicit role for policy interventions. In Section 3 we briefly consider the acyclical equilibrium growth path and Section 4 we develop the cyclical equilibrium growth path. Section 5 considers the welfare implications of switching between the two equilibria and Section 6 discusses the consequences of various stabilization policies. Section 7 concludes and mathematical details are relegated to an appendix.

## 2 The Model

### 2.1 Assumptions

There is no aggregate uncertainty. Time is continuous and indexed by  $t \geq 0$ . The economy is closed. The representative household has iso-elastic preferences

$$U(t) = \int_t^\infty e^{-\rho(\tau-t)} \frac{C(\tau)^{1-\sigma} - 1}{1-\sigma} d\tau, \quad (1)$$

where  $\rho$  denotes the rate of time preference and  $\sigma$  represents the inverse of the elasticity of intertemporal substitution. The household maximizes (1) subject to the intertemporal budget constraint

$$\int_t^\infty e^{-[R(\tau)-R(t)]} C(\tau) d\tau \leq S(t) + \int_t^\infty e^{-[R(\tau)-R(t)]} [w(\tau) + \Upsilon(\tau)] d\tau \quad (2)$$

where  $w(t)$  denotes wage income,  $S(t)$  denotes the household's stock of assets (firm shares and capital) at time  $t$  and  $R(t)$  denotes the discount factor from time zero to  $t$ . The term  $\Upsilon(\tau)$  represents lump-sum transfers from the government (see below). The population is normalized to unity and each household is endowed with one unit of labor hours, which it supplies inelastically.

Final output is produced according to a Cobb-Douglas production function utilizing physical capital,  $K(t)$ , and a continuum of intermediates,  $x_i$ , indexed by  $i \in [0, 1]$ :

$$Y(t) = K(t)^\alpha X(t)^{1-\alpha}, \quad (3)$$

where  $\alpha \in (0, 1)$  and

$$X(t) = \exp \int_0^1 \ln x_i(t) di. \quad (4)$$

Final output can be used for private consumption,  $C(t)$ , investment,  $\dot{K}(t)$ , or (potentially) stored. Moreover, household may receive transfers net of all tax revenue from the government amounting to  $\Psi(t)$ . It follows that

$$C(t) + \dot{K}(t) + \delta K(t) \leq Y(t) + \Psi(t), \quad (5)$$

where  $\delta$  denotes the rate of physical depreciation. Although we allow physical capital to be reversible in principle, in the equilibria we study negative investment never actually occurs.

Output of intermediate  $i$  depends upon the state of technology in sector  $i$ ,  $A_i(t)$ , and labor hours,  $L_i(t)$ , according to a simple linear technology:

$$x_i(t) = A_i(t)L_i(t) \tag{6}$$

Intermediates are completely used up in production, but can be produced and stored for later use. Incumbent intermediate producers must therefore decide whether to sell now, or store and sell later.

Commercially viable productivity improvements are introduced into the economy via a process of “entrepreneurial search”. Competitive entrepreneurs in each sector allocate labor effort to searching for ideas, and finance this by selling claims. The rate of success from search is  $\mu h_i(t)$ , where  $\mu$  is a parameter, and  $h_i$  represents the labor effort allocated to search in sector  $i$ . At each date, entrepreneurs decide whether or not to allocate labor to search, and if they do so, how much. The aggregate labor effort allocated to search is given by

$$H(t) = \int_0^1 h_i(t) dt. \tag{7}$$

New ideas and innovations dominate old ones in terms of productivity by a factor  $e^\gamma$ , where  $\gamma > 0$ . This process is therefore formally identical to the innovation process in the quality-ladder model of Grossman and Helpman (1991). However, we explicitly do not interpret this activity as R&D. Although it is common to do so in the endogenous growth literature, this Poisson process is, in fact, a very bad description of R&D. Typically R&D is a knowledge-intensive (and often capital-intensive) activity, which involves accumulation of sector-specific knowledge. In sharp contrast, the search activity described here is a skill-intensive one, which we interpret as a form of entrepreneurship. In our view this entrepreneurial function is the central player in economic activity, with R&D playing a supportive role that is not modeled here.<sup>2</sup> This activity could be undertaken by independent entrepreneurs, but in modern production it is often a role taken on by managers and other skilled workers **within** firms.<sup>3</sup>

Successful manager/entrepreneurs must choose whether or not to implement commercially viable ideas immediately or delay until a later date. Once they implement, the associated knowledge becomes publicly available, and can be built upon by rivals. However, prior to implementation, the knowledge is privately held by the entrepreneur. We let the indicator function  $Z_i(t)$  take on the value 1 if there exists a commercially viable innovation in sector  $i$  which has not yet been implemented, and 0 otherwise. The set of dates in which new ideas are implemented in sector  $i$  is denoted by  $\Omega_i$ . We let  $V_i^I(t)$  denote the expected present

---

<sup>2</sup>In Francois and Lloyd-Ellis (2006b) we introduce endogenous R&D as a separate, knowledge-intensive activity that generates ideas whose commercial viability is unclear. As in the current paper, entrepreneurial search is counter-cyclical, but R&D investment is pro-cyclical.

<sup>3</sup>For example, Nickell, Nicolitsas and Patterson (2001) find that “managerial innovations” are concentrated during in downturns.

value of profits from implementing a success at time  $t$ , and  $V_i^D(t)$  denote that of delaying implementation from time  $t$  until the most profitable future date.

## 2.2 The Public Sector

We consider a limited number of simple policy instruments. Profits earned from intermediate production may be taxed/subsidized at a constant rate,  $\omega \in (-1, 1)$ .<sup>4</sup> The return to capital can be taxed/subsidized at a rate  $\tau(t) \in (-1, 1)$ , which could be time-varying. In particular, we will allow this tax rate to vary between the expansions and contractions. We assume throughout that depreciation is tax-deductible. Total revenue from these proportional taxes is denoted by  $\Phi(t)$

As noted above, the government can also levy lump-sum taxes and pay out lump-sum transfers, where the net transfer at time  $t$  is denoted  $\Upsilon(t)$ . The net outlay of the public sector at time  $t$  are therefore given by  $\Psi(t) = \Upsilon(t) - \Phi(t)$ . In the equilibrium analysis below, we assume throughout that the public sector satisfies its intertemporal budget constraint

$$\int_t^\infty e^{-[R(s)-R(t)]} \Psi(s) ds = \int_t^\infty e^{-[R(s)-R(t)]} [\Upsilon(s) - \Phi(s)] ds = 0, \quad (8)$$

where without loss of generality we assume that the government's initial asset position is zero.

Before proceeding to analysis of the welfare, variance and growth effects of government interventions, we first define an equilibrium in this model and then characterize the stationary – acyclical – equilibrium which is analogous to the standard equilibrium of Schumpeterian quality ladders models. We then proceed to solve for the more complicated cyclical equilibrium, which involves a stationary cycle.

## 2.3 Definition of Equilibrium

Given initial state variables  $\{A_i(0), Z_i(0)\}_{i=0}^1, K(0)$  an equilibrium for this economy consists of:

(1) sequences  $\hat{p}_i(t), \hat{x}_i(t), \hat{L}_i(t), \hat{h}_i(t), \hat{A}_i(t), \hat{Z}_i(t), \hat{V}_i^I(t), \hat{V}_i^D(t)_{t \in [0, \infty)}$  for each intermediate sector  $i$ , and

(2) economy wide sequences  $\hat{Y}(t), \hat{K}(t), \hat{R}(t), \hat{w}(t), \hat{q}(t), \hat{C}(t), \hat{S}(t)_{t \in [0, \infty)}$

which satisfy the following conditions:

- Households allocate consumption over time to maximize (1) subject to the budget constraint, (2). The first-order conditions of the household's optimization imply that

$$\hat{C}(t)^\sigma = \hat{C}(\tau)^\sigma e^{\mathbf{R}(t) - \mathbf{R}(\tau) - \rho(t-\tau)} \quad \forall t, \tau, \quad (9)$$

<sup>4</sup>The profit tax can be thought of as a convenient representation of various other realistic factors, including implementation costs, imitation and labor market imperfections, which are not directly modeled in our set up, but which imply that some of the rent accruing to firms is not appropriated by their owners. Since the full amount of this revenue is not appropriated by the government, it is not able to set this tax to zero. However, it can influence the level.

and that the transversality condition holds

$$\lim_{\tau \rightarrow \infty} e^{-\hat{R}(\tau)} \hat{S}(\tau) = 0 \quad (10)$$

- Final goods producers choose capital and intermediates,  $x_i$ , to minimize costs given prices  $p_i$ , subject to (3). The derived demand for intermediate  $i$  is

$$x_i^d(t) = (1 - \alpha) \frac{Y(t)}{p_i(t)}. \quad (11)$$

The conditional demand for capital is given by

$$K(t) = \frac{\alpha Y(t)}{q(t)} \quad (12)$$

- The unit elasticity of demand for intermediates implies that limit pricing at the unit cost of the previous incumbent is optimal. It follows that

$$p_i(t) = \frac{w(t)}{e^{-\gamma} A_i(t)} \quad \forall t \quad (13)$$

The resulting instantaneous profit (before any taxes) earned in each sector is given by

$$\pi(t) = (1 - e^{-\gamma})(1 - \alpha)Y(t). \quad (14)$$

- Labor markets clear:

$$\int_0^1 \hat{L}_i(t) di + \hat{H}(t) = 1 \quad (15)$$

- Arbitrage trading in financial markets implies that, for all assets that are held in strictly positive amounts by households, the rate of return between time  $t$  and time  $s$  must equal  $\frac{\hat{R}(s) - \hat{R}(t)}{s - t}$ .
- Free entry into entrepreneurship:

$$\mu \max[\hat{V}_i^D(t), \hat{V}_i^I(t)] \leq \hat{w}(t), \quad \hat{h}_i(t) \geq 0 \quad \text{with at least one equality.} \quad (16)$$

- At dates where there is implementation, entrepreneurs with commercially viable ideas must prefer to implement rather than delay until a later date

$$\hat{V}_i^I(t) \geq \hat{V}_i^D(t) \quad \forall t \in \hat{\Omega}_i. \quad (17)$$

- At dates where there is no implementation, either there must be no useful ideas available to implement, or entrepreneurs with successful ideas must prefer to delay rather than implement:

$$\begin{aligned} \text{Either } \hat{Z}_i(t) &= 0, \\ \text{or if } \hat{Z}_i(t) &= 1, \hat{V}_i^I(t) \leq \hat{V}_i^D(t) \quad \forall t \notin \hat{\Omega}_i. \end{aligned} \quad (18)$$

- Free entry of replacement capital.
- The government's intertemporal budget constraint holds.



### 3 The Acyclical Equilibrium Growth Path

In the economy we have described, there exists an acyclical steady state equilibrium growth path which closely corresponds to that already well analyzed in Schumpeterian quality ladders models. Along such a path, entrepreneurial successes are implemented immediately, and due to the large number of sectors, the economy grows smoothly at rate  $g^a$ , with a symmetric allocation of entrepreneurial effort to all sectors. There are no fluctuations in any of the economy's aggregates. Provided that the parameters of the model are such that  $r(t) > g^a(t)$  along the equilibrium path, so that household utility is bounded, then the growth rate is given by

$$g^a = \frac{\gamma [\mu(1 - e^{-\gamma})(1 - \omega) - \rho e^{-\gamma}]}{(\sigma - 1)\gamma + 1} \quad (19)$$

This equilibrium describes an economy that is, in most respects, identical to that which has been analyzed in Grossman and Helpman (1991), with the exception of the complications arising from capital accumulation.<sup>5</sup> Note that the long-run growth rate in this equilibrium is independent of capital taxes,  $\tau(t)$ , and net transfers,  $\Upsilon(t)$ , and depends only on the tax/subsidy on profits,  $\omega$ . In this model, capital accumulation plays only a supporting role in driving long-run growth and, while an increase in  $\tau$  will effect the level of  $K(t)$  along the balanced growth path, it does not impact the long-run growth rate. Changes in public net transfers have no distortionary affects and, again, only affect levels not growth rates. In contrast, increasing  $\omega$  directly reduces the marginal returns to innovation causing growth to fall. These same qualitative implications for long-run growth also carry over to the cyclical equilibrium discussed below. However, as we will see, all three of these policy parameters have short-run growth affects and hence have implications for volatility and welfare.

### 4 The Cyclical Equilibrium Growth Path

Suppose that implementation occurs at discrete dates denoted by  $T_\nu$  where  $\nu \in \{1, 2, \dots, \infty\}$ . We adopt the convention that the  $\nu$ th cycle starts at time  $T_{\nu-1}$  and ends at time  $T_\nu$ . After implementation at date  $T_{\nu-1}$  an expansion is triggered by a productivity boom and continues through subsequent fixed capital formation. During this phase entrepreneurial search ceases and consequently all labor effort is used in production. At some time  $T_\nu^*$ , search commences and labor starts to be withdrawn from production. Commercially viable ideas are not implemented immediately but are withheld until time  $T_\nu$ . During this contraction phase, fixed capital formation continues, but the rate of investment declines rapidly. As aggregate demand falls, labor continues to be released from production, so that search accelerates in anticipation of the subsequent implementation boom.

---

<sup>5</sup>Aghion and Howitt (2003, ch. 3) develop a related Schumpeterian growth model which allows for capital accumulation. In these models, capital accumulation has little impact on the qualitative nature of the growth path.

Let  $P_i(s)$  denote the probability that, since time  $T_{v-1}$ , no commercially viable ideas have materialized in sector  $i$  by time  $s$ . It follows that the probability of there being no success by time  $T_v$  conditional on there having been none by time  $t$ , is given by  $P_i(T_v)/P_i(t)$ . Hence, the value of an incumbent firm in a sector where new ideas have arisen by time  $t$  during the  $v$ th cycle can be expressed as

$$V_{0,i}^I(t) = (1 - \omega) \int_t^{T_v} e^{-[R(\tau) - R(t)]} \pi_i(\tau) d\tau + \frac{P_i(T_v)}{P_i(t)} e^{-\beta(t)} V_{0,i}^I(T_v). \quad (20)$$

where

$$\beta(t) = R_0(T_v) - R(t) \quad (21)$$

denotes the discount factor used to discount from time  $t$  during the cycle to the beginning of the next cycle.<sup>6</sup> The first term in (20) represents the discounted profit stream that accrues to incumbent firms with certainty during the current cycle, and the second term is the expected discounted value of being an incumbent at the beginning of the next cycle. In a cyclical equilibrium, the identification of commercially viable ideas can be credibly signalled immediately and all search in their sector stops until the next cycle (see Lemma 2 in the Appendix). Unsuccessful entrepreneurs have no incentive to falsely announce search success. As a result, an entrepreneur's signal is credible, and other entrepreneurs will exert their efforts in sectors where they have a better chance of becoming the dominant entrepreneur.

In the cyclical equilibrium, entrepreneurs' conjectures ensure no more entrepreneurship in a sector once a signal of success has been received, until after the next implementation. The time  $t \in (T_v^*, T_v)$  expected value of a viable idea whose implementation is delayed until time  $T_v$  is thus:

$$V_i^D(t) = e^{-\beta(t)} V_{0,i}^I(T_v). \quad (22)$$

In the cyclical equilibrium, such delay is optimal; i.e.  $V_i^D(t) > V_i^I(t)$  throughout the contraction. Successful entrepreneurs are happier to forego immediate profits and delay implementation until the boom in order to ensure a longer reign of incumbency. Since no implementation occurs during the cycle, by delaying, firms are assured of incumbency until at least  $T_{v+1}$ . Incumbency beyond that time depends on the probability that another viable ideas is identified.

The symmetry of sectors implies that search effort is allocated evenly over all sectors that have not yet experienced a success within the cycle. In the posited cyclical equilibrium, the probability of not being displaced at the next implementation is

$$P_i(T_v) = \exp \left[ -\mu \int_{T_v^*}^{T_v} h_i(\tau) d\tau \right]. \quad (23)$$

---

<sup>6</sup>Throughout, we use the subscript 0 to denote the value of a variable immediately after the boom. Formally, for any variable  $X(\cdot)$ , we define  $X(t) = \lim_{\tau \rightarrow t^-} X(\tau)$  and  $X_0(t) = \lim_{\tau \rightarrow t^+} X(\tau)$ .

## 4.1 Within-Cycle Dynamics

In equilibrium, factor prices are proportional to their marginal products. Consequently, standard aggregation results hold and aggregate output can be expressed as

$$Y(t) = \bar{A}_{v-1}^{1-\alpha} K(t)^\alpha L(t)^{1-\alpha}, \quad (24)$$

where

$$\bar{A}_{v-1} = \exp \int_0^1 \ln A_i(T_{v-1}) di. \quad (25)$$

Note that this endogenous component of TFP is fixed through the cycle. In order to afford a stationary representation of the economy it is convenient to normalize aggregates by dividing by total factor productivity using lower-case letters to denote these deflated aggregates:

$$k(t) = \frac{K(t)}{\bar{A}_{v-1}}, \quad c(t) = \frac{C(t)}{\bar{A}_{v-1}}, \quad y(t) = \frac{Y(t)}{\bar{A}_{v-1}}, \quad \psi(t) = \frac{\Psi(t)}{\bar{A}_{v-1}} \quad (26)$$

Consequently, the intensive form production function is given by

$$y(t) = k(t)^\alpha L(t)^{1-\alpha}. \quad (27)$$

The household's Euler equation during the cycle can be expressed as

$$\frac{\dot{c}(t)}{c(t)} = \frac{r(t) - \rho}{\sigma}, \quad (28)$$

where  $r(t) = \dot{R}(t)$ . The economy's aggregate resource constraint is

$$\dot{k}(t) = y(t) - c(t) + \psi(t) - \delta k(t). \quad (29)$$

Finally, factor prices can be expressed as

$$q(t) = \alpha k(t)^{\alpha-1} L(t)^{1-\alpha} \quad (30)$$

$$w(t) = e^{-\gamma} (1 - \alpha) \bar{A}_{v-1} k(t)^\alpha L(t)^{-\alpha}. \quad (31)$$

Note that the wage rate is less than its marginal product by a factor  $e^{-\gamma}$ , reflecting the fact that a fraction  $1 - e^{-\gamma}$  goes in the form of profits to intermediate producers. Moreover, free entry of replacement capital implies that

$$r(t) = (1 - \tau^i) (q(t) - \delta), \quad (32)$$

where the capital tax rate is allowed to vary between expansions ( $i = X$ ) and contractions ( $i = C$ ).

#### 4.1.1 The Expansion ( $T_{v-1} \rightarrow T_v^*$ )

We now trace out the evolution of the economy implied by the behavior posited above. We start immediately following an implementation boom, when capital, consumption and output take on the initial values  $k_0(T_{v-1})$ ,  $c_0(T_{v-1})$  and  $y_0(T_{v-1})$ , respectively. During the expansion all labor is used in production so that

$$L(t) = 1. \quad (33)$$

Combining this condition with (27), (28), (29), (30) and (32) yields transitional dynamics that are identical to those of the Ramsey model:

$$\frac{\dot{c}(t)}{c(t)} = \frac{(1 - \tau^X)\alpha k(t)^{\alpha-1} - \delta - \rho}{\sigma} \quad (34)$$

$$\frac{\dot{k}(t)}{k(t)} = k(t)^\alpha - c(t) - \delta k(t) + \psi(t). \quad (35)$$

These dynamics are illustrated using a phase diagram in Figure 1.

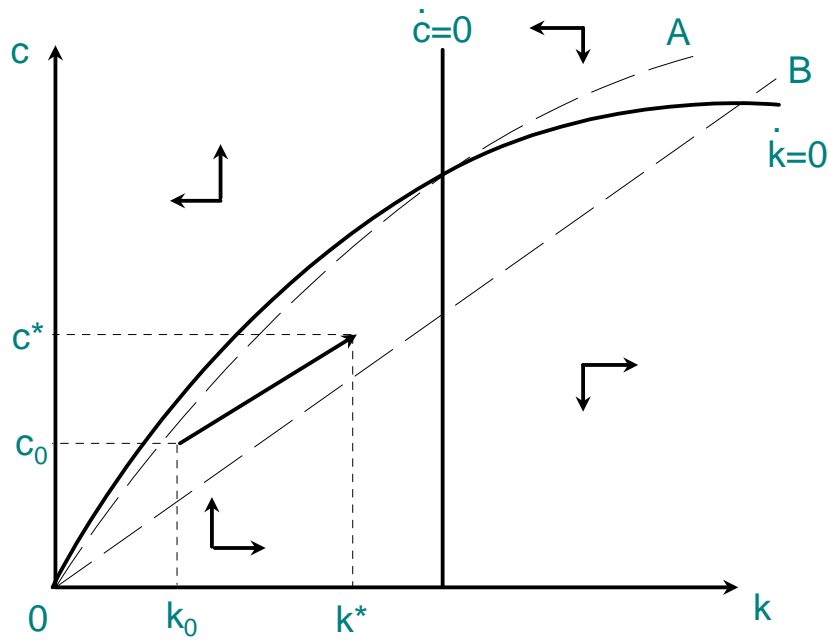


Figure 1: Dynamics in Phase 1

During this expansionary phase, both consumption and capital grow, so we restrict attention to the lower left quadrant of the phase diagram. As capital accumulates, the wage grows and the interest rate declines. However, for the dynamic path to be consistent with the cyclical equilibrium, the dynamic path must also lie between the curves  $OA$  and  $OB$  in Figure 1. During the first phase of the cycle, entrepreneurial

search with delayed implementation cannot be optimal. That is

$$\mu V^D(t) < w(t). \quad (36)$$

As the capital stock accumulates and TFP grows,  $w(t)$  rises through time. Moreover, as the subsequent boom approaches  $V^D(t)$  grows at the rate of interest. As long as the path of the economy lies between  $OA$  and  $OB$  it must be true that

$$r(t) > \frac{\dot{w}(t)}{w(t)}. \quad (37)$$

Consequently, the first phase of the cycle comes to an end in finite time.

After the date,  $T_v^*$ , if all labor were to remain in production, returns to search effort would strictly dominate those in production. As a result, labor effort is re-allocated from production and into search and this triggers the next phase of the cycle. During the transition from one phase to the next, all aggregate variables evolve smoothly (see Lemma ??).

#### 4.1.2 The Downturn ( $T_v^* \rightarrow T_v$ )

During this phase, capital continues to be accumulated so that (32) must still hold. However, now there is search, so that  $L(t) < 1$ . Free entry into entrepreneurship implies  $\mu V^D(t) = w(t)$ , so that it must be the case that

$$r(t) = \frac{\dot{V}^D(t)}{V^D(t)} = \frac{\dot{w}(t)}{w(t)}. \quad (38)$$

Combining these conditions with (27), (28), (29), (30) and (31) implies that during the slowdown, consumption, capital and the labor force in production evolve according to the following dynamical system:

$$\frac{\dot{c}(t)}{c(t)} = \frac{(1 - \tau^C) \alpha k(t)^{\alpha-1} L(t)^{1-\alpha} - \delta - \rho}{\sigma} \quad (39)$$

$$\dot{k}(t) = k(t)^\alpha L(t)^{1-\alpha} - c(t) - \delta k(t) + \psi(t) \quad (40)$$

$$\frac{\dot{L}(t)}{L(t)} = -\frac{c(t) + \psi(t)}{k(t)} + \frac{1 - \alpha}{\alpha} (\delta + \tau^C) k(t)^{\alpha-1} L(t)^{1-\alpha} - \frac{\delta}{\alpha} \quad (41)$$

The value of  $\dot{L}(t)/L(t)$  is negative during this phase and is ensured by condition (63). Initially, the consumption-capital ratio  $c(t)/k(t)$  also continues to decline. However, as  $L(t)$  declines, the marginal product of capital falls and investment starts to fall. Eventually, in the hypothesized cycle,  $c(t)/k(t)$  starts to rise again. Note that the variable  $G(t)$ , government expenditure in the recession, enters here to lower capital accumulation, and labor adjustment in the recession. This shall be one of the policies focused on in the policy analysis.

Note that the implied path for  $L(t)$  during this phase implies a path for the fraction of the labor effort engaged in search,  $H(t) = 1 - L(t)$ . This, in turn, determines the measure of sectors in which commercially viable ideas are identified at each date:

$$-\dot{P}(t) = \mu [1 - L(t)], \quad (42)$$

where  $P(T_v^*) = 1$ .<sup>7</sup> At the end of the cycle, the fraction of sectors that have experienced successful search is therefore

$$1 - P(T_v) = \int_{T_v^*}^{T_v} \mu [1 - L(\tau)] d\tau. \quad (43)$$

### 4.1.3 The Implementation Boom

We denote the improvement in total factor productivity during implementation,  $e^{(1-\alpha)\Gamma_v}$ , where  $\Gamma_v = \ln \frac{\bar{A}_v}{\bar{A}_{v-1}}$ . Productivity growth at the boom is given by

$$\Gamma_v = \gamma(1 - P(T_v)). \quad (44)$$

A key implication of the assumption that investment is (at least partially) reversible is that household consumption must evolve smoothly over the period  $T_v$  — it cannot jump discontinuously. Intuitively, if households anticipated a sharp rise in consumption in the future they could raise their utility by converting some of the capital stock into consumption goods immediately. As a result, the household's Euler equation implies that the rate of return on any asset held over the boom must equal zero.<sup>8</sup> In particular, the return to storing intermediate goods until after the boom must be zero. A positive return would exist if the wage rose discontinuously upon implementation because it would be cheaper to produce extra intermediates at the low wage just before the boom and substitute them for production at the high wage afterwards. The fact that, in equilibrium, the wage must therefore evolve smoothly across the boom pins down a tight relationship between the growth in productivity and the labor effort allocated back into production:

$$(1 - \alpha)\Gamma_v = -\alpha \ln L(T_v) \quad (45)$$

During the boom, firm values and wages grow in proportion to labor productivity. Since, just before the boom  $\mu V^I(T_v) = w(T_v)$ , an immediate corollary is that

$$\mu V_0^I(T_v) = w_0(T_v) = (1 - \alpha)e^{-\gamma} \bar{A}_v k_0(T_v)^\alpha. \quad (46)$$

Output growth through the boom is given by

$$\Delta \ln Y(T_v) = (1 - \alpha)\Gamma_v - (1 - \alpha) \ln L(T_v) = \frac{1 - \alpha}{\alpha} \Gamma_v \quad (47)$$

It follows directly from that growth in output exceeds the discount factor across the boom. Since profits are proportional to output, this explains why firms are willing to delay implementation during the downturn. Because investment is reversible, consumption cannot jump at the boom, and so all of the increase in output is associated with a sharp rise in investment.

---

<sup>7</sup>The rate of change in  $P$  is given by  $\frac{\dot{P}}{P} = -\mu h_i$ . But since labor is allocated symmetrically to innovation only in the measure  $P$  of sectors where no innovation has occurred,  $h_i = \frac{H}{P}$ , so that  $\dot{P} = -\mu H$ .

<sup>8</sup>This is in stark contrast to Shleifer (1986) and Francois and Lloyd-Ellis (2003), where consumption jumps at the boom.

## 4.2 The Stationary Cyclical Equilibrium

We focus on a stationary cyclical equilibrium growth path in which the boom size is constant at  $\Gamma$  every cycle and the cycle length is given by  $\Delta = T_v - T_{v-1}$ . In addition, we denote the length of the stationary expansion phase as  $\Delta^* = T_v^* - T_{v-1}$ . Through this cycle, optimal entrepreneurial behaviour must satisfy following requirements:

- At time  $t = T_v$ , firms must prefer to implement immediately, rather than delay implementation until later in the cycle or the beginning of the next cycle:

$$V_0^I(T_v) > V_0^D(T_v). \quad (\text{E1})$$

- In sectors where viable ideas are identified during the downturn, firms must prefer to wait until the beginning of the next cycle rather than implement earlier and sell at the limit price:

$$V^I(t) < V^D(t) \quad \forall t \in (T_v^*, T_v) \quad (\text{E2})$$

- Search is not optimal during the expansion of the cycle. Since in this phase of the cycle  $\mu V^D(t) < w(t)$ , this condition requires that

$$\mu V^I(t) < w(t) \quad \forall t \in (T_{v-1}, T_v^*) \quad (\text{E3})$$

- Finally, in constructing the equilibrium above, we have implicitly imposed the requirement that the downturn is not so long that viable ideas are identified in every sector:

$$P(T_v) > 0. \quad (\text{E4})$$

In Francois and Lloyd-Ellis (2006a) we show that under fairly weak parametric restrictions, only one pair of values  $\hat{\Gamma}, \hat{\Delta}$  can satisfy conditions (E1) to (E4) along a stationary cyclical path.<sup>9</sup>

## 4.3 Simulated Aggregate Behavior

As the economy's evolution is dictated by dynamic paths for capital, consumption, investment and entrepreneurial effort, that are the solutions to differential equations, it is not possible to solve for the behavior of this economy analytically. We therefore simulate the economy for a benchmark set of parameter values and sketch the evolution of aggregates through the cycle.<sup>10</sup> The exercise also suffices to establish existence of the cycle. We take as a benchmark the following sets of parameter values:

---

<sup>9</sup>This does not imply that the equilibrium is globally unique, only that it is so within the class of stationary cyclical paths described above. We know that there exists at least one other equilibrium growth path — the standard acyclical one.

<sup>10</sup>The simulations are performed in Gauss, and are available from the authors upon request.

Table 1: Baseline Calibration

Parameter	$\alpha$	$\gamma$	$\rho$	$\sigma$	$\mu$	$\delta$	$\omega$
Value	0.30	0.20	0.025	1.00	1.80	0.10	0.70

The parameters  $\alpha$  and  $\gamma$  imply a capital share of 0.3, a labor share of about 0.6, and a profit share of 0.1. The unit-valued intertemporal elasticity of substitution implies logarithmic preferences (following Lucas, 1987). Given these values and a depreciation rate of 10%, we chose  $\mu$ ,  $\rho$  and  $\omega$  so as to match a long-run annual growth rate of almost 2%, an average risk-free real interest rate of roughly 4%, and a cycle length of approximately 8 years. These values roughly correspond to average data for the post-war US. The implied value of  $\omega$  is admittedly rather high if we interpret it purely as a tax on profits. However, as noted earlier, we view  $\omega$  as representing a number factors that may affect the ratio of profits to wages (e.g. labor market frictions, implementation costs, imitation). For the benchmark parameters a cycle with average growth of 1.9% per annum is obtained.

Figure 2 plots the paths of the main aggregates: consumption, investment and income, over the cycle. Aggregate GDP expands for just over 5 years before declining for the remainder of the cycle, and then booming dramatically just before the end of the eighth year. Despite the considerable volatility of output, and, to a lesser extent GDP, capital markets allow consumers to maintain a relatively smooth consumption path with the bulk of fluctuations occurring in investment. Note that consumption growth does vary over the cycle, even becoming negative during the last year but, as in the data, its variation is small compared to investment. The difference between GDP and aggregate income only arises in the economy's recessionary phase and happens because GDP includes the value of entrepreneurial effort allocated to the search for improvements as well as measured output or aggregate income.

## 5 The Welfare Benefit of Cycles

In the spirit of Lucas (1993) and Barlevy (2004b), we limit our analysis to the study of steady state growth paths, without considering transition paths between steady states. Consequently, the policy experiments performed here are not computations of the overall welfare consequences of policy variations or changes in expectations starting from a particular steady state. Such analysis would involve computing transitional dynamics and would undermine comparability with earlier papers. Instead, what we report here are the welfare implications of removing households from an economy evolving along one stationary growth path and dropping them into one evolving along another, holding constant their initial asset position.

When comparing steady-state welfare along the steady-state growth path of any endogenous growth model with two asset stocks, a complication arises because the steady state growth path is always associated with a particular ratio of these state variables. If a policy variation changes this steady-state ratio, initial



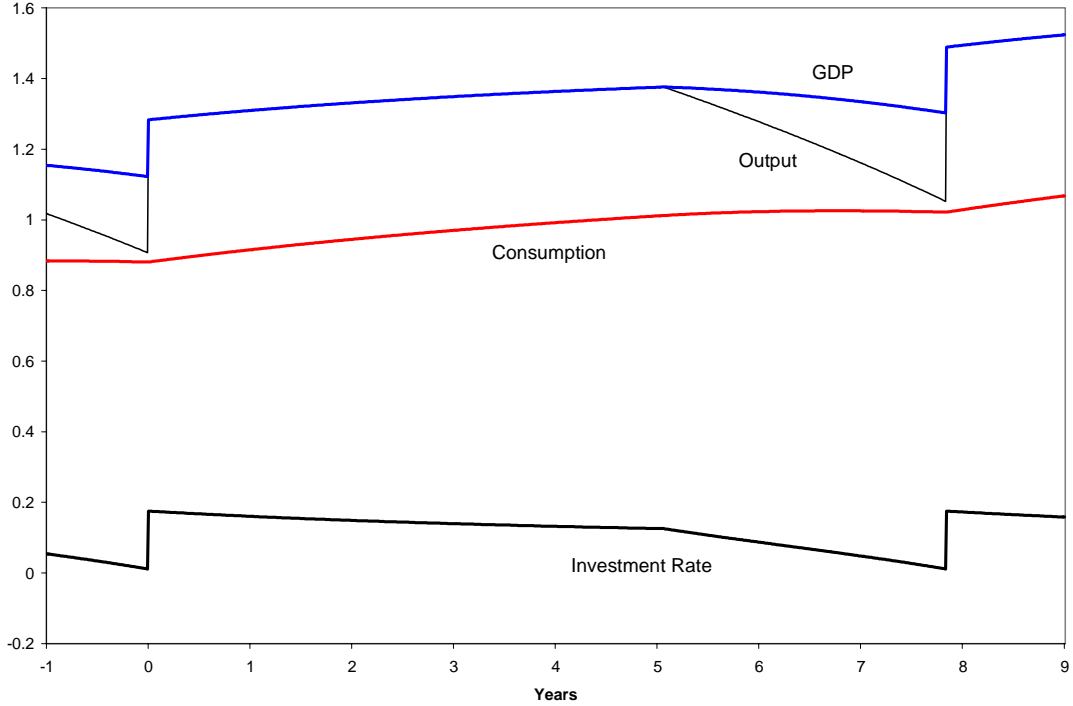


Figure 2: Aggregate Variables

conditions must change. Along the acyclical long-run growth path of the current model, for example, the ratio  $k = K/\bar{A}$  takes on a constant, particular value for each set of parameters. This makes it impossible to derive a meaningful comparison of steady-state welfare under alternative policy scenarios because initial values of both  $K$  and  $\bar{A}$  cannot held constant.

In the cyclical steady state here, however, such a comparison is feasible because  $k$  varies over the cycle. In what follows, we normalize the initial value of  $\bar{A} = 1$  and choose the date  $t_0 \in (0, \Delta)$  at which to start computing welfare, so that  $k(t_0) = \hat{k}$  across all parameter settings. This ensures that steady-state welfare for each policy scenario and for each equilibrium is computed from identical starting values of the state variables. The welfare gains/losses that we measure therefore reflect only the impact of changes in long-run growth and the shape of cycles, and not changes in the initial values of capital and TFP.

### 5.0.1 Welfare Calculations

The representative agent's welfare, assuming log utility, in the cycling economy at time  $t_0$  is given by:

$$W(t_0) = \frac{\Gamma e^{-\rho\Delta}}{\rho(1 - e^{-\rho\Delta})} + \frac{\int_{t_0}^{t_0+\Delta} e^{-\rho(t-t_0)} \ln C(t) dt}{1 - e^{-\rho\Delta}} \quad (48)$$

It is also shown there that, in the acyclical economy, welfare is given by:

$$W^a(t_0) = \frac{1}{\rho} \ln C(t_0) + \frac{g_a}{\rho^2} \quad (49)$$

$$C(t_0) = \frac{\rho + g_a}{\alpha(1 - \tau)} + \frac{1 - \alpha}{\alpha} (\delta - g_a) K(t_0). \quad (50)$$

As the simulations above indicate, since consumers are able to smooth consumption through the capital market, fluctuations in output are unlikely to have major impacts on welfare directly. However, as we shall see below, government policies that aim to alter such fluctuations can have large effects on welfare indirectly through their impact on the economy’s average growth rate.<sup>11</sup>

## 5.1 Cyclical and Acyclical Comparison

Before turning to a comparison of outcomes under different policies, we compare welfare and growth in the cycling and acyclical equilibria. Strictly speaking this is not a policy choice, unless it is possible for governments to coordinate agents’ expectations on the equilibrium, but it is of interest to consider it here as it allows direct comparison with the literature that grew out of Lucas’ original comparison.

Table 2 compares the cycling and acyclical steady states for the benchmark parameter values. Computing welfare in the cycling economy at the benchmark  $k = 1.977$  implies measuring from a point 0.693 years (8 months) into the expansion.<sup>12</sup> We normalize welfare in this benchmark case at 1 and measure welfare in all other scenarios relative to this value. For this baseline calibration, steady state welfare is over 30% higher in the cyclical equilibrium than the acyclical one. The reason for this is that the cycling economy is able to sustain a considerably higher average growth rate. The higher variance in the growth rate of output has little direct impact on welfare because consumption evolves relatively smoothly (as in the data).

Table 2: Cyclical vs. Acyclical

	Cyclical	Acyclical
Average Growth Rate (%)	1.90	1.50
Variance of Growth Rate (%)	0.05	0.00
Relative Welfare (log utility)	1.00	0.68

To understand why growth is higher in the cyclical equilibrium, suppose (hypothetically) that future growth along the two paths were expected to be the same. Along the acyclical growth path, productivity improvements are implemented immediately, so that the opportunity cost (due to lost production) of

<sup>11</sup>We have conducted the welfare analyses with  $\sigma > 1$  (and which satisfy existence conditions). This makes little difference to the main results.

<sup>12</sup>We choose this benchmark because it corresponds to the capital stock at the boom in one of the higher capital simulations to follow. Choosing a different benchmark does not effect the qualitative nature of the comparison.

searching for new ideas rises in proportion to the growth in their value, with no reallocation of labor effort across sectors. During the downturn of a cyclical equilibrium implementation is delayed, so that the opportunity cost would not rise in proportion to their value unless search intensity increases. Consequently, in equilibrium, innovative effort is high and rising during downturns. During expansions, following implementation, the opportunity cost of search rises so much relative to the benefits that to equate them would require more labor effort to be allocated into production than is available. Since search effort is bounded below by zero, it follows that the average level of entrepreneurship would be higher in the cyclical economy for the same expected future growth. Since higher expected future growth increases the incentives for innovation in both equilibria, the actual long-run growth rate must be higher in the cyclical one.

Since the reasoning underlying the higher growth rate in the cycling economy is generically true for any cycling equilibrium, the result is robust to parameter variations. It stands in stark contrast to the majority of the literature which has, for the most part, argued that the welfare implications of fluctuations are small and negative. Here they are large and positive.

## 6 Impact of Stabilization Policies

### 6.1 Tax on Profits

In this policy experiment, we set the capital tax rate equal to zero and consider variations in the tax rate on profits. We assume that tax revenue is immediately transferred back to households in a lump-sum fashion, so that  $\psi(t) = 0$  and

$$\Upsilon(t) = \omega\pi(t) \quad \forall t. \tag{51}$$

The table below documents the implications of varying the tax on profits,  $\omega$ . The benchmark where  $\omega = 0.7$  is highlighted in bold.

Table 3: Variation in the Profit Tax

$\omega$ value	Welfare	$\bar{g}(\%)$	$\text{Var}(g)$	Expansion	Contraction	Acyclic Welfare	Acyclic $g$
0.50	1.94	3.3	0.120	2.98	2.91	1.61	2.8
0.55	1.80	3.0	0.100	3.20	2.89	1.38	2.5
0.60	1.57	2.6	0.091	3.70	2.87	1.14	2.2
0.65	1.20	2.2	0.069	4.23	2.84	0.91	1.9
0.70	<b>1.00</b>	<b>1.9</b>	<b>0.056</b>	<b>5.03</b>	<b>2.82</b>	<b>0.68</b>	<b>1.5</b>
0.75	0.83	1.6	0.041	5.92	2.81	0.44	1.2
0.80	0.56	1.3	0.030	8.45	2.78	0.21	0.9

At a superficial level, such taxes appear to have favorable stabilization properties. The variance of the growth rate falls with  $\omega$  and the economy spends longer in expansionary phases, and less time in recessions. Comparing the two extreme values reported in the table. The economy with  $\omega = 0.5$  spends nearly half its

time in recession, whereas that with  $\omega = 0.8$  is in expansion over 75% of the time. The high tax economy has much less variability in its growth rate – the variance of growth in the  $\omega = 0.5$  economy is four times that in the  $\omega = 0.8$  economy, and its contractions are shorter and less severe (output falls by less). In short, raising profit taxes would seem an ideal countercyclical policy.

But the value of  $\omega$  also affects average long-run growth,  $\bar{g}$ . As profits are the inducement to innovation, the post-tax value of entrepreneurship is lower with higher  $\omega$  and entrepreneurial levels are correspondingly lower, so that growth rates are too. The intuition for this is similar to that which occurs when profit taxes are levied in the acyclical economy, as confirmed by the final column; by lowering net returns to entrepreneurship, both it and the growth rate fall.

The reason for the smoothing effects of such taxes is less obvious. As returns to entrepreneurship are lower, it takes longer for the value of returns to entrepreneurial effort to approach those to effort allocated into production in the cycling economy. Consequently, the economy spends longer in the expansionary phase. But since the expansionary phase is not one in which efforts are allocated to future productivity improvements, growth rates fall. A lower growth rate is also consistent with a smaller contractionary phase. The relationship between the size of the jump in output at the boom,  $\Gamma$ , and the allocation of efforts to entrepreneurship is tightly pinned down by the no arbitrage condition over the boom. As is therefore clear from the welfare results in the first column, the smoothing effects of such taxes come at considerable cost to consumer welfare.

## 6.2 Tax/Subsidy on Capital

We now fix  $\omega = 0.7$  and consider variations in the tax rate on capital. As before we assume that tax revenue is immediately transferred back to households in a lump-sum fashion, so that  $\psi(t) = 0$  and

$$\Upsilon(t) = \tau^i [q(t) - \delta] K(t) + \omega\pi(t) \quad \forall t \quad (52)$$

Table 4 documents the results for a range of values of the capital income tax that is levied in the expansionary phase  $\tau^X(t)$  and the contractionary phase  $\tau^C(t)$ . We allow for both positive (taxes) and negative (subsidies) values of the variable, and we again list the baseline values in bold. As in the acyclical equilibrium, average long-run growth is unaffected by these alternatives. The tax combinations do affect phase lengths, particularly expansions. However, these are of little consequence for the representative household — although steady-state welfare does vary because of differences in the path of consumption over the cycle, the effects are tiny because long-run growth of consumption is the dominant factor.

Taxing capital returns in the expansion,  $\tau^X > 0$ , lengthens the expansion. On the one hand, the increase in the cost of accumulating capital lowers the average capital stock that is held over the cycle, but since net accumulation occurs through the expansionary phase, as the rate of accumulation is reduced, more time is required for the capital stock to accumulate to the point where it matches the increased

productivity arising from the boom. Subsidizing capital accumulation in the contraction,  $\tau^C < 0$ , has a qualitatively similar, though less pronounced, effect. The expansion increases by a year when  $\tau^X = .1$ , but by less than six months for  $\tau^C = -0.1$ . The subsidizing of capital accumulation in the contraction has this effect even though it has the opposite consequence of increasing the average level of capital holdings through the cycle. The subsidy to accumulation in recession makes consumers willing to hold a larger stock of capital on average, but since net accumulation of capital occurs through the expansion — when the cost of capital is relatively low due to the implementation boom — the expansionary phase must last longer. Thus, although these taxes have substantial effects on volatility as they shift the pattern of accumulation through the cycle, the welfare impacts are small.

Table 4: Variation in the Capital Tax

$\tau^X$	$\tau^C$	Var( $g$ )	Expansion	Contraction
0.15	0.00	0.065	6.59	2.95
0.10	0.00	0.061	6.04	2.90
0.05	0.00	0.054	5.49	2.86
0.00	0.00	0.056	5.03	2.82
-0.05	0.00	0.049	4.81	2.79
-0.10	0.00	0.048	4.38	2.76
0.00	-0.10	0.053	5.44	2.81
0.00	-0.05	0.058	5.06	2.82
0.00	0.00	0.056	5.03	2.82
0.00	0.05	0.051	4.87	2.82
0.00	0.10	0.052	4.62	2.83
0.00	0.15	0.048	4.61	2.83

### 6.3 Intertemporal Reallocation

The policy variations discussed above both assume that the government balances its budget every period. Here we consider a tax–transfer scheme that reallocates resources between the two phases of the cycle. Specifically we assume

$$\Psi(t) = \Upsilon^i - \omega\pi(t) \neq 0 \tag{53}$$

where  $i \in \{X, C\}$  and

$$\int_{T_{v-1}}^{T_v^*} e^{-[R(t)-R(T_{v-1})]}\psi^X dt + \int_{T_v^*}^{T_v} e^{-[R(t)-R(T_{v-1})]}\psi^C dt = 0. \tag{54}$$

Table 5 documents various the implications of various combinations of net transfers for the steady state cyclical path. In each case we fix a value for  $\psi^X$  and then set  $\psi^C$  so that (54) holds in equilibrium. We consider both countercyclical and pro–cyclical reallocations. In the benchmark calibration, GDP averages about  $1.25 \times \bar{A}_v$  over the cycle (see Figure 2), so the numbers in the first two columns can be interpreted as representing something close to fractions of GDP (the federal primary surplus averages around 2%).

These intertemporal re-allocations have no impact on long-run growth (growth remains at its benchmark value of 1.9%). On the other hand, as can be seen, countercyclical tax/transfer policies can be used to substantially reduce the variance of output growth. Transferring resources from the expansion to the recession reduces the marginal utility cost of allocating labour to search. Search activity starts earlier, causing the expansion to become shorter. The resulting increase in search activity at each date after  $T^*$  also causes both the recession length to contract and the boom size to decline. All of these effects contribute to lower volatility. Once again, however, the welfare impacts of the countercyclical reallocation are negligible, mainly because households are able to undo such lump-sum reallocations via their savings behaviour and because there are no long-run growth benefits.

Table 5: Intertemporal Reallocation via Lump-Sum Transfers

$\psi^X$	$\psi^C$	Var( $g$ )	Expansion	Contraction	$\Gamma$
0.02	-0.054	0.070	6.77	3.03	0.189
0.01	-0.023	0.066	5.63	2.91	0.168
0.00	0.00	0.056	5.03	2.82	0.152
-0.01	0.0200	0.049	4.60	2.76	0.138
-0.02	0.0360	0.044	4.12	2.69	0.126
-0.03	0.0485	0.041	3.70	2.65	0.121
-0.04	0.0590	0.040	3.43	2.61	0.115
-0.05	0.0685	0.034	3.12	2.58	0.108

## 7 Conclusions

We consider an economy in which cycles are the equilibrium outcome of a Schumpeterian endogenous growth process. Allowing the cycle to be endogenously determined forces us to explicitly recognize that it may not be possible to target one aspect of the economy's evolution, say the volatility of output, or the length of contractions, without affecting other components, such as the growth rate. To our knowledge, the analysis we have performed here is the first one which examines the general equilibrium effects of counter-cyclical policies on both growth rates and cyclical features.<sup>13</sup> We used our framework to analyze the welfare impact of (1) eradicating cycles by switching to another equilibrium growth path, and (2) policies designed to mitigate fluctuations along the cyclical growth path.

Because contractions enable entrepreneurs to allocate labour effort towards productivity-improving activities, they serve an important role here. This is reflected in the paper's two main findings. Firstly, the cyclical economy devotes more resources to growth promoting activities on average than does the acyclical one. Consequently, average growth rates are higher in the cyclical economy. Secondly, policies designed to mitigate contractions and reduce the volatility of output, will often come at the cost of reducing the

<sup>13</sup>Francois and Shi (1999) note the potential for implementation cycles to increase growth, but their model does not permit a welfare analysis.

economy's average growth rate. The overall conclusions here are thus quite different to those usually reported in the welfare costs of business cycles literature.

One reason for the stark contrast between our results and those reported by Barlevy (2004a) is that, in the Schumpeterian world described here, capital does not play a first-order important role in the growth process. According to Schumpeter (1950), entrepreneurs are the key movers of productivity, and this is an idea-intensive (not capital-intensive) process. Fixed capital formation is required to offset depreciation, and to augment capital stocks in response to productivity growth, but it plays a largely auxiliary role. Smoothing the investment rate yields no benefits for long-term growth. In contrast, smoothing the allocation of entrepreneurial efforts can damage the growth generation process.

In the analysis conducted here, we have abstracted from several potentially important issues in order to emphasize the potential role of endogenous cyclical variation. For example, variations in involuntary unemployment during recessions might have additional welfare costs that are not being considered here. Moreover, the cycle that we study here is deterministic and stationary. Variations in the aggregate growth rate and the length and amplitude of the cycle could be introduced by adding a exogenous fluctuations in productivity growth. Introducing features such as these would quantitatively affect our results, but we believe the qualitative implications would continue to hold.

## Appendix

Proposition 1: If  $\mu\gamma(1 - e^{-\gamma})(1 - \omega) + \rho(\gamma(1 - e^{-\gamma}) + 1/(\sigma - 1)) > 0$ , then there exists an acyclical equilibrium with a constant growth rate given by

$$g^a = \max \left\{ \frac{\gamma[\mu(1 - e^{-\gamma})(1 - \omega) - \rho e^{-\gamma}]}{(\sigma - 1)\gamma + 1}, 0 \right\}. \quad (55)$$

Proof: Since  $x_i$  and  $p_i$  are proportional to  $A_i$  it is easy to verify that solutions are symmetric across sectors:  $K_i = K$ , and  $L_i = L$  for all  $i$ , with  $w$  given by:

$$w(t) = (1 - \alpha)e^{-\gamma}y(t)/(1 - H(t)). \quad (56)$$

Since implementation is immediate, the aggregate rate of endogenous productivity growth is

$$g(t) = \mu\gamma H(t) \quad (57)$$

No-arbitrage implies that

$$r(t) + \mu H(t) = \frac{(1 - \omega)\pi(t)}{V^I(t)} + \frac{\dot{V}^I(t)}{V^I(t)} \quad (58)$$

Since, innovation occurs in every period, free entry into entrepreneurship implies that

$$\mu V^I(t) = w(t). \quad (59)$$

Along the balanced growth path, all aggregates grow at the rate  $g$ . From the Euler equation it follows that

$$r(t) = \rho + \sigma g. \quad (60)$$

Along the BGP,  $H(t) = H$  is constant. Differentiating (56) and (59) w.r.t. to time yields

$$\frac{\dot{V}^I(t)}{V^I(t)} = g + \phi \quad (61)$$

using this to substitute for  $\frac{\dot{V}^I(t)}{V^I(t)}$  in (58), and using (60) to substitute for  $r(t)$  and (14) to substitute for  $\pi(t)$ , and solving for  $g$  yields (55). For the consumer's utility to be bounded we need  $r = \rho + \sigma g > g$  so that:

$$\rho + (\sigma - 1) \frac{\gamma [\mu(1 - e^{-\gamma})(1 - \omega) - \rho e^{-\gamma}]^{\frac{1}{\sigma}}}{(\sigma - 1)\gamma + 1} > 0. \quad (62)$$

Re-arranging yields the condition in the proposition.  $\text{\textcancel{Y}}$

**Lemma 1:** In a cyclical equilibrium, the identification of commercially viable ideas can be credibly signalled immediately and all search in their sector stops until the next round of implementation.

**Proof:** We show: (1) that if a signal of success from a potential entrepreneur is credible, other entrepreneurs stop innovation in that sector; (2) given (1), entrepreneurs have no incentive to falsely claim success.

**Part (1):** If entrepreneur  $i$ 's signal of success is credible then all other entrepreneurs believe that  $i$  has a productivity advantage which is  $e^\gamma$  times better than the existing incumbent. If continuing to innovate in that sector, another entrepreneur will, with positive probability, also develop a productive advantage of  $e^\gamma$ . Such an innovation yields expected profit of 0, since, in developing their improvement, they do not observe the non-implemented improvements of others, so that both firms Bertrand compete with the same technology. Returns to attempting innovation in another sector where there has been no signal of success, or from simply working in production,  $w(t) > 0$ , are thus strictly higher.

**Part (2):** If success signals are credible, entrepreneurs know that upon success, further innovation in their sector will cease, from Part (1), by their sending of a costless signal. They are thus indifferent between falsely signalling success when it has not arrived, and sending no signal. Thus, there exists a signalling equilibrium in which only successful entrepreneurs send a signal of success.  $\text{\textcancel{Y}}$

**Proposition 2:** If, during an expansion, the dynamic paths of consumption and capital satisfy

$$\left(1 - \frac{\alpha}{\sigma}\right) k(t)^\alpha + \frac{\mu}{\sigma} \frac{\rho + (1 - \sigma)\delta}{\sigma} k(t) > c(t) > \frac{\mu}{\alpha} (1 - \alpha) \delta k(t), \quad (63)$$

then there exists a  $T_v^*$  such that for the first time in a given cycle  $\mu V^D(T_v^*) = w(T_v^*)$ .



Proof: Subtracting (35) from (34) we get

$$\frac{\dot{c}(t)}{c(t)} - \frac{\dot{k}(t)}{k(t)} = \frac{c(t)}{k(t)} - \left[ 1 - \frac{\alpha}{\sigma} \right] k(t)^{\alpha-1} - \frac{\rho + (1-\sigma)\delta}{\sigma} \quad (64)$$

It follows that in order for  $c(t)/k(t)$  to be declining in the first phase

$$\frac{c(t)}{k(t)} < \left[ 1 - \frac{\alpha}{\sigma} \right] k(t)^{\alpha-1} + \frac{\rho + (1-\sigma)\delta}{\sigma}. \quad (65)$$

which is the left hand inequality in (63). For  $r > \dot{w}/w$ , we require that

$$\alpha k(t)^{\alpha-1} - \delta > \alpha \frac{\dot{k}(t)}{k(t)} \quad (66)$$

Substitution using (35) yields

$$\alpha k(t)^{\alpha-1} > \delta + \alpha \left[ k(t)^{\alpha-1} - \frac{c(t)}{k(t)} - \delta \right], \quad (67)$$

which rearranges to the right hand inequality in (63).  $\sphericalangle$

Lemma 2: At time  $T_v^*$ , when entrepreneurial search first commences in a cycle,  $L(T_v^*) = 1$  and output, investment and consumption must evolve continuously.

Proof: Since capital depreciation rates are independent of utilization, and its marginal product is always positive, installed capital is always fully utilized. At  $T_v^*$ , since the discount factor does not jump, neither does  $V^D$ . With non-variable capital utilization, the wage must jump up if  $L(t)$  jumps down. Since just before  $T_v^*$ ,  $\mu V^D(t) < w(t)$ , this is not possible. It follows that  $L(T_v^*) = 1$  and falls smoothly from that point on.

Since  $L$  adjusts smoothly, and capital utilization is non-variable, output cannot jump down at  $T_v^*$ . Since the discount factor does not jump, consumption cannot jump at  $T_v^*$  either. Consequently, investment,  $\dot{K}$ , cannot jump at  $T_v^*$ . Note further that  $r(t) = q(t) - \delta$  cannot jump down instantaneously with non-variable capital utilization. It follows that wage growth  $\frac{\dot{w}(t)}{w(t)}$  must jump up at  $T_v^*$  and employment growth in production  $\frac{\dot{L}(t)}{L(t)}$  must jump to a negative level.  $\sphericalangle$

Proposition 3: Asset market clearing under reversible investment at the boom requires that

$$(1 - \alpha)\Gamma_v = -\alpha \ln L(T_v).$$

Proof: During the boom, for entrepreneurs to prefer to implement immediately, it must be the case that  $V_0^I(T_v) > V_0^D(T_v)$ . Just prior to the boom, when the probability of displacement is negligible, the value of implementing immediately must equal that of delaying until the boom:  $\delta V^I(T_v) = \delta V^D(T_v) = w(T_v)$ . Free entry into entrepreneurship at the boom requires that  $\delta V_0^I(T_v) \leq w_0(T_v)$ . The opportunity cost of financing entrepreneurship is the rate of return on shares in incumbent firms in sectors where no innovation

has occurred. Since this return across the boom must equal zero, it must be the case that  $V_0^I(T_v) = V^I(T_v)$ .

It follows that asset market clearing at the boom requires

$$\log \frac{w_0(T_v)}{w(T_v)} = (1 - \alpha)\Gamma_v - \ln L(T_v) \geq 0. \quad (68)$$

In sectors with no entrepreneurial success, incumbent firms could sell claims to stored output, use them to finance greater current production and then store the good to sell at the beginning of the next boom. Free entry into storage implies that the rate of return (the growth in the wage) to this activity must satisfy

$$\log \frac{w_0(T_v)}{w(T_v)} = (1 - \alpha)\Gamma_v - \ln L(T_v) \leq 0. \quad (69)$$

Combining (68) and (69) yields (45).

**Welfare Calculations (log utility):** Welfare in the cyclical economy can be expressed as

$$\begin{aligned} W &= \int_0^\infty e^{-\rho t} \ln c(t) dt = \int_{v=0}^\infty e^{-\rho \Delta v} \int_0^\Delta e^{-\rho t} [\Gamma v + \ln c(t)] dt \\ &= \Gamma \frac{1 - e^{-\rho \Delta}}{\rho} \int_{v=0}^\infty e^{-\rho \Delta v} v + \int_{v=0}^\infty e^{-\rho \Delta v} \int_0^\Delta e^{-\rho t} \ln c(t) dt \\ &= \Gamma \frac{1 - e^{-\rho \Delta}}{\rho} \frac{e^{-\rho \Delta}}{(1 - e^{-\rho \Delta})^2} + \frac{\int_0^\Delta e^{-\rho t} \ln c(t) dt}{1 - e^{-\rho \Delta}} \\ &= \frac{\Gamma e^{-\rho \Delta}}{\rho(1 - e^{-\rho \Delta})} + \frac{\int_0^\Delta e^{-\rho t} \ln c(t) dt}{1 - e^{-\rho \Delta}} \end{aligned}$$

Welfare in the acyclically growing economy is given by

$$\begin{aligned} W^a(0) &= \int_0^\infty e^{-\rho t} \ln C(t) dt = \int_0^\infty e^{-\rho t} [\ln C(0) + g_a t] dt \\ &= \frac{1}{\rho} \ln C(0) + \int_0^\infty e^{-\rho t} g_a t dt = \frac{1}{\rho} \ln C(0) - \frac{1}{\rho} \int_0^\infty g_a t d e^{-\rho t} \\ &= \frac{1}{\rho} \ln C(0) - \frac{1}{\rho} g_a t e^{-\rho t} \Big|_0^\infty + \frac{g_a}{\rho} \int_0^\infty e^{-\rho t} dt \\ &= \frac{1}{\rho} \ln C(0) + \frac{g_a}{\rho^2} \end{aligned}$$

## References

- Aghion, Philippe and Howitt, Peter, "A Model of Growth Through Creative Destruction," *Econometrica*, vol. 60, 1992, pp. 323–351.
- Barlevy, Gadi (2004a) "The Cost of Business Cycles Under Endogenous Growth," *American Economic Review*, vol. 94(4), pages 964-990.
- Barlevy, Gadi (2004b) "The Cost of Business Cycles and the Benefits of Stabilization: A Survey," NBER working paper 10926..
- Dolmas, J. (1998) Risk preferences and the welfare cost of business cycles, *Review of Economic Dynamics* 1: 646-676
- Francois, P. and H. Lloyd–Ellis, (2003) "Animal Spirits Through Creative Destruction," *American Economic Review*, vol. 93 (3), June, pp. 530-550.
- Francois, P. and H. Lloyd–Ellis (2006a) "I-Q Cycles," mimeo Queen's University.
- Francois, P. and H. Lloyd–Ellis (2006b) "Schumpeterian Cycles with Pro–Cyclical R&D," mimeo Queen's University.
- Francois, P. and S. Shi (1999) "Innovation, Growth and Welfare Improving Cycles", *Journal of Economic Theory*, 85, 226-257.
- Gale, Douglas (1996) "Delay and Cycles," *Review of Economic Studies*, vol. 63, pp. 169-198.
- Imrohoroglu, A. (1989) Cost of business cycles with indivisibilities , *Journal of Political Economy* 97: 1364-1383
- Lucas, Robert, (1987) *Models of Business Cycles*. Oxford: Basil Blackwell
- Obstfeld, M. (1994) Evaluating risky consumption paths: the role of intertemporal substitutability, *European Economic Review* 38: 1471-1486.
- Pemberton, J. (1996) Growth trends, cyclical fluctuations, and welfare with non- expected utility preferences , *Economics Letters* 50:387-392
- Schumpeter, Joseph, *Capitalism, Socialism and Democracy*, 3rd edition, New York, 1950.
- Shleifer, Andrei, "Implementation Cycles" *Journal of Political Economy*, vol. 94 (6), 1986, pp. 1163–1190.
- Storesletten, Kjetil, Chris I. Telmer and Amir Yaron (2000) The Welfare Cost of Business Cycles Revisited: Finite Lives and Cyclical Variation in Idiosyncratic Risk NBER working paper 8040.
- Van Wincoop, E. (1994) Welfare gains from international risk sharing , *Journal of Monetary Economics* 34:175-200.

