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Should wages be subsidized in a pandemic?

Brant Abbott

Nam Phan

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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Brant Abbott
Queen's University
abbottbrant@gmail.com

Nam Van Phan
Queen's University
phan.nam1009@gmail.com

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Abstract. We use a labor search model with heterogenous households and firms to study the efficacy of a wage subsidy during a pandemic, relative to enhancing unemployment benefits. A large proportion of the economy is forced to shut down, and firms in that sector choose whether to lay off workers or keep them on payroll. A wage subsidy encourages firms to keep workers on payroll, which speeds up labor market recovery after the pandemic ends. However, a wage subsidy can be costlier than enhancing unemployment benefits. If the shutdown is long or profit margins are low then a wage subsidy is preferable, and vice-versa. The optimal mixture of policies includes a wage subsidy that covers 90% the first $200/week of earnings, and expands unemployment benefits to cover all salary up to $275/week. Low income workers, as well as those in less productive jobs, benefit the most from a wage subsidy.

JEL Classification: E2, E32, J40
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1 Introduction

Various employment related policies and programs were implemented in response to the COVID-19 pandemic. These can be broadly characterized as (i) enhancing unemployment insurance programs, and (ii) directly subsidizing wage and salary payments. Many countries implemented programs with elements of both (i) and (ii). In the United States, large unemployment benefit and direct transfer programs were enacted, but not a formal wage subsidy. However, the Paycheck Protection Program included the subsidy-like characteristic of loan forgiveness if employees were not laid off. Canada, on the other hand, introduced an explicit wage subsidy, which paid 75% of the wages of workers in affected firms, up to a ceiling. At the same time, Canada expanded its unemployment benefit programs. In Australia, affected firms were reimbursed for wage payments to their workers, up to a limit. These workers might have been laid off otherwise and claimed unemployment benefits. Indeed, the vast majority of OECD countries introduced either expanded unemployment benefits, a wage subsidy program, or both.[1]

An important question to then consider is whether wage subsidies paid to firms are more effective than direct transfers to affected workers (unemployment benefits). The most pressing objective of these policies is to maintain household incomes at a time when people cannot work, and both types of policies accomplish this. An added benefit of a wage subsidy program is described in the legislation enacting Australia's program: "By temporarily offsetting wage costs, the JobKeeper scheme supports businesses to retain staff and continue paying them despite suffering decreased turnover during this period of downturn. The payment also supports these businesses to recommence their operations or scale up operations quickly without needing to rehire when the downturn is over."[2] Put differently, a wage subsidy ensures that worker-firm connections are maintained, speeding up economic recovery after the pandemic ends because a period of high unemployment due to search frictions is avoided. A downside of a wage subsidy, however, is that firms likely anticipate this feature of a recovery, and so might have kept their workers on payroll anyway, or have kept in close contact with laid-off workers in order to recall them. Thus, some workers wages might be unnecessarily subsidized, which is fiscally costly.[3] In deciding on the optimal policy mix, a planner must weigh all of these tradeoffs in the generosity of a wage subsidy, and at the same time consider unemployment benefits as an alternative.

In this paper we develop a model of the economic transitions from a pandemic shock through to full economic recovery. The model includes an endogenous distribution of consumption and

[1] See https://www.imf.org/en/Topics/imf-and-covid19/Policy-Responses-to-COVID-19 for the IMF's country-by-country description of COVID-19 economic policies.

[2] https://www.legislation.gov.au/Details/F2020L00419/Explanatory%20Statement/Text

[3] Lutz, Cho, Crane *et al.* (2020) find that the Paycheck Protection Program, for example, had only a limited effect in protecting jobs.

savings, heterogeneity in both worker and job productivity, and a frictional labor market. The model is used to study the effects of alternative pandemic-era wage subsidy and unemployment benefit programs on the labor market and household welfare. The analysis accounts for short-run effects during the pandemic, the medium-term recovery, and long-run effects through the fiscal implications of alternative policies. We find that (1) it is optimal is to subsidize 85% of the wages of affected workers up to a $350 per week ceiling, when unemployment benefits are held constant. (2) However, enhanced unemployment insurance with a 100% replacement rate for workers up to $275 per week ceiling in affected sectors is better than the optimal wage subsidy if the shutdown duration is less than 11 weeks, or if the mean profit margin of affected firms is larger than 12.25%. (3) The biggest winners from a wage subsidy are workers with little liquidity (credit), and workers who would otherwise have been laid off in the absence of the wage subsidy. These features are correlated with income, hence low-income workers benefit the most from the wage subsidy, while workers earning more than $1200 per week experience a welfare loss under the optimal wage subsidy scheme. (4) There are substantial welfare gains from implementing a wage subsidy and enhanced unemployment benefits jointly, with the optimal policy being a 90% subsidy up to $200/week along with up to $275/week of added unemployment benefits.

Our model builds upon Krusell, Mukoyama, and Şahin (2010) (KMS) in several ways.[4] First, we introduce fixed heterogeneity in job-specific productivity and labor productivity. This allows us to study how pandemic policies affect different types of workers and jobs. Second, we allow firms to choose to lay off workers. In normal times this feature does not matter because all jobs generate positive value; however, during the pandemic many jobs have negative value because of production stoppages, so endogenous layoffs do then occur. We allow firms to possibly recall laid off workers, although this option disappears over time. This recall feature is motivated by Fujita and Moscarini (2017) who show that a substantial share of workers return to their previous employer after a jobless spell. Third, we assume that there is no bargaining when a match between a worker and firm is formed, and instead wages are fixed in a way that ensures matches are always desirable during normal times. We make this assumption based on the insight of Hall (2005) that a DMP model with fixed wages generates realistic employment fluctuations.[5] In this way our model can be thought of as a partial-equilibrium extension of KMS.

The model is ideal for studying the long-run consequences of a short-run disruption in labor markets because it naturally accommodates the welfare effects of the disruption itself, as well as the long-run fiscal costs of related policies. Our model captures the economic disruption as a

[4] Krusell, Mukoyama, and Şahin (2010) (KMS) combine an incomplete-markets model (e.g. Huggett (1993) or Aiyagari (1994)) with a frictional DMP labor market model (e.g. Diamond (1982), Mortensen and Pissarides (1994), or Pissarides (2000))

[5] Abstracting from bargaining reduces the state space of a worker, as their wage would otherwise depend on their wealth level at the time they become employed, and thus would need to be tracked.

period in which some firms are forced to suspend production. Firms choose whether or not to lay off their employees in response to this. We assume that a worker who is laid-off because of the shutdown remains attached to their firm, and, unless this attachment dissolves, they will be recalled when it is profitable for the firm to do so. A firm's layoff decision depends on match productivity. For a high productivity match, the firm will choose to keep their worker on payroll, despite the shutdown, because they anticipate a return to highly profitable production in the future. However, the opposite is true for a low productivity match.

With the introduction of a wage subsidy, the threshold for a match to be productive enough to keep a worker on payroll is lowered so that fewer workers are laid off. One benefit of using the wage subsidy to lower the productivity threshold, and thus cause more matches to be retained, is that these jobs immediately restart production when the lockdown ends. Another benefit is to maintain the income of the worker. Enhanced unemployment benefits also maintain worker income, but some laid-off workers will not return to their previous job. The longer a shutdown goes on for, the smaller the number of laid-off workers who are still attached to their previous job and can be recalled. The cost of either policy is that tax rates must be higher in the future, which lowers the welfare of all taxpayers. A wage subsidy might be more expensive than enhanced unemployment benefits because some firms end up being subsidized even though they would not lay off workers even in the absence of a subsidy.

There are many examples of past research that has incorporated the DMP framework into fully-specified macro models that account for business cycle fluctuations. The theoretical side of such models was first investigated by Howitt (1988) and Wright (1986), and the quantitative implications were studied by Andolfatto (1996). Shimer (2005) then argued that the standard DMP model cannot generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies as observed in the US. Since then, several papers have modified the DMP framework with the intention of solving Shimer's puzzle. Examples include Hall (2005), Hagedorn and Manovskii (2008), Pissarides (2009), and Gertler and Trigari (2009), among others. In a related strand of literature, Krusell, Mukoyama, Şahin *et al.* (2009) and Krusell, Mukoyama, and Şahin (2010) study aggregate welfare under various counterfactual policy experiments within the KMS framework.

There is also a large literature studying the impact of government policies during the pandemic. The most related of these studies is Birinci, Karahan, Mercan *et al.* (2021), who also compare an unemployment benefit expansion to a payroll subsidy. The primary difference between their work and our own is that our model features an endogenous distribution of wealth, whereas their model assumes households are hand-to-mouth. We are able to analyze how the benefits and costs of the two policy options vary over the wealth distribution, and indeed find that there are winners and losers under the optimal policy. Our work is also quite related to Giupponi and Landais (2020) and Giupponi, Landais, Lapeyre *et al.* (2021), who analyze the question of whether policy makers should be concerned with maintaining employment or worker incomes during downturns. Although these latter papers are not directly related to a pandemic, their conclusion that unemployment insurance is complementary to short-time-work, which is similar to a wage subsidy program, is consistent with our own conclusions.

Interactions between the pandemic and labor markets are also studied by Boar and Mongey (2020), Fang, Nie, and Xie (2020), Gregory, Menzio, and Wiczer (2020) and Mitman and Rabinovich (2020). Other work on the macroeconomics of a pandemic includes Eichenbaum, Rebelo, and Trabandt (2020), Faria-e Castro (2021), Kapicka and Rupert (2020), Bayer, Born, Luetticke *et al.* (2020), and Kaplan, Moll, and Violante (2020), among others, who integrate the Susceptible-Infectious-Recovered (SIR) framework into economic models to study the impact of the COVID-19 pandemic on the economy, and the extent to which fiscal policies (such as transfers or debt forgiveness) could alleviate the economics damage. Other related work include Acemoglu, Chernozhukov, Werning *et al.* (2020) and Glover, Heathcote, Krueger *et al.* (2020), who develop an overlapping generation model integrated with SIR to study the optimal lockdown and the associated distributional consequence for different age cohorts in the economy. Inoue, Murase, and Todo (2020) and Barrot, Grassi, and Sauvagnat (2020) use a production network model to study the impact of a partial economic lockdown and social distancing on the production of all sectors.

2 Model

We study an environment where the economy starts in a steady-state equilibrium. We then shock the economy by forcing production to cease in a large part of the economy for a period of time, interpreted as the pandemic period, after which production resumes as normal. However, the economy does not immediately return to the pre-pandemic steady-state, but rather follows a (potentially lengthy) transition path to a new steady-state. Because the government offers income replacement and wage subsidy programs during the pandemic, the new steady-state will include higher labor income taxes in order to pay back debt accumulated during the shutdown period.

We present a detailed description of the initial steady-state equilibrium before moving on to the pandemic shock.

2.1 Initial Steady State

2.1.1 Preliminaries

Time is continuous and indexed by t. Agents living in the initial steady-state believe this equilibrium will continue forever, and thus have no foresight about the pandemic. The economy is populated by a unit continuum of workers who live forever. There is also a mass of firms who can post job vacancies and produce when matched with a worker.

Worker Preferences: Each worker has an expected utility function $\mathbb{E}_0\left[\int_{t=0}^{\infty} e^{-\rho t} v(c(t)) dt\right]$, where c is time-varying consumption, and $v(c) = c^{1-\varsigma}/(1-\varsigma)$ is the flow utility function with coefficient of relative risk aversion ς. The discount rate, ρ, is heterogeneous in the population, taking one of three values $\rho \in \{\rho_1, \rho_2, \rho_3\}$. A worker's ρ value is random, but persistent, with the common rate of transition to a new discount rate denoted by π_ρ. Because the transition rate is the same for all values of ρ, in aggregate the discount rate is uniformly distributed across the three support values.

Firm Technology and Payoffs: When a firm matches with a worker a match-specific productivity z is drawn. The output of a firm is the product of this draw and the productivity of their worker, θ. Workers are paid a wage equal to their productivity, and firms also pay other costs, such as capital depreciation expenses and overhead, which are a proportion β of their revenue. Combined, these features imply that a firm's profit flow is $z\theta - \theta - \beta z\theta$. We assume the distribution of z is such that $(1-\beta)z > 1$. Firms discount their profit flow at rate R, and this flow ends at the rate of exogenous separation occurrence, σ. A firm that does not have a worker may pay a flow-cost κ of posting a vacancy in order to search for a worker.

Labor Market: The measure of unemployed workers at a point in time is denoted U, and the measure of job vacancies is denoted ν. The measure of employed workers is $E = 1 - U$. Vacant jobs and unemployed workers are randomly matched according a constant-returns-to-scale (CRS) matching function $M(U, \nu)$. Through this technology, workers find jobs at rate f and vacancies are filled at rate q, according to

$$\begin{aligned} f &= \frac{M(U,\nu)}{U} = M(1,\Theta) \\ q &= \frac{M(U,\nu)}{\nu} = M(\frac{1}{\Theta},1) \end{aligned} \tag{1}$$

where $\Theta \equiv \nu/U$ is the market tightness. From a firm's perspective the skill of a matched worker is random, and the match-specific productivity of a job is drawn from a conditional distribution $P_z(z|\theta)$ after the match occurs.

Financing: We assume an exogenous real interest rate r, which is the return on wealth for workers with positive net worth. Workers are able to borrow up to a limit $\underline{b}$, but pay a different interest rate R than savers earn. We assume an exogenous financing wedge is responsible for this difference, hence $R > r$. This cost of borrowing is also the discount rate of firms. Firms are owned by exogenous entities, thus households only invest in them indirectly through their saving.

Government: Government levies a progressive tax on worker earnings described by the tax function $T(y)$. Associated revenue is used to finance an unemployment insurance program, which pays $h(\theta)$ to an unemployed worker whose usual earnings are θ. Remaining government revenue is used to pay for non-valued government expenditure G, thus the government budget constraint holds every period. These policy parameters are exogenous to the model, thus the government does not make any decisions in the initial steady-state.

2.1.2 Workers

Workers choose their consumption to maximize their utility, subject to the law of motion for their wealth: $\dot{b} = y + r(b)b - c$. Worker net income is denoted y, and the interest rate they face is denoted $r(b)$, which depends whether the worker is a borrower or saver ($r(b) = R$ if $b < 0$ and $r(b) = r$ if $b \geq 0$). Worker consumption-savings decisions are subject to a borrowing limit $b \geq \underline{b}$.

A worker's income depends on their fixed skill level θ, as well as their employment status $s \in \{e, u\}$, where e denotes employed and u denotes unemployed. Net income in the two employment states is summarized as follows:

$$y = \begin{cases} \theta - T(\theta) & \text{if} \quad s = e \\ h(\theta) & \text{if} \quad s = u. \end{cases} \tag{2}$$

A worker's flow of income when employed is their productivity θ, less their associated flow of tax payments $T(\theta)$.

The optimal consumption-savings decisions of employed workers solve the following HJB equation:

$$\begin{aligned} \rho_i V(i,b,\theta,e) = & \max_c v(c) + \partial_b V(i,b,\theta,e)\dot{b} + \sigma\big(V(i,b,\theta,u) - V(i,b,\theta,e)\big) \\ & + \sum_{j \neq i} \pi_\rho \big(V(j,b,\theta,e) - V(i,b,\theta,e)\big) \\ \text{s.t.} \ \ & \dot{b} = \theta - T(\theta) + r(b)b - c, \ \ b \geq \underline{b}. \end{aligned} \tag{3}$$

Here $V(i,b,\theta,e)$ is the value function of an employed worker with discount rate ρ_i, wealth b and productivity (wage rate) θ. At rate π_ρ the worker's discount rate will jump from ρ_i to ρ_j, and an associated jump in the value function will occur.[6] Similarly, at the rate of exogenous worker firm separation, σ, the value function jumps to that of an unemployed worker $V(i,b,\theta,u)$. Workers who

[6] We assume that transitions between any levels ρ_i and ρ_j have the same rate of occurrence, π_ρ, which ensures a uniform stationary distribution of types.

are exogenously separated from their employer cannot be recalled, hence the unemployed worker value function depends on the job finding rate, as follows:

$$\begin{aligned}\rho_i V(i,b,\theta,u) = &\max_c v(c) + \partial_b V(i,b,\theta,u)\dot{b} + f\big(V(i,b,\theta,e) - V(i,b,\theta,u)\big)\\ &+ \sum_{j\neq i} \pi_\rho\big(V(j,b,\theta,u) - V(i,b,\theta,u)\big)\\ \text{s.t. } \dot{b} = &\, h(\theta) + r(b)b - c, \quad b \geq \underline{b}.\end{aligned} \tag{4}$$

The savings decisions in the problems above, along with the employment fluctuations that arise in labor market equilibrium, generate the endogenous distribution of wealth in the model. This joint distribution of wealth and other state variables, denoted $g(i,b,\theta,s)$, evolves according to a Kolmogorov Forward Equation (KFE), which is separated into parts due to employed and unemployed workers, as follows:

$$\begin{aligned}\dot{g}(i,b,\theta,e) = &- \partial_b[\dot{b}(i,b,\theta,e)g(i,b,\theta,e)] + fg(i,b,\theta,u) - \sigma g(i,b,\theta,e)\\ &+ \sum_{j\neq i} \pi_\rho\big(g(j,b,\theta,e) - g(i,b,\theta,e)\big)\\ \dot{g}(i,b,\theta,u) = &- \partial_b[\dot{b}(i,b,\theta,u)g(i,b,\theta,u)] - fg(i,b,\theta,u) + \sigma g(i,b,\theta,e)\\ &+ \sum_{j\neq i} \pi_\rho\big(g(j,b,\theta,u) - g(i,b,\theta,u)\big).\end{aligned} \tag{5}$$

The first line on the RHS of each equation is standard, while the second line of each RHS results from the inclusion of time-varying discount factors. In a steady-state equilibrium the time derivatives of the distribution (the LHS) will be required to be zero.

2.1.3 Firms:

The value of a job with match-specific productivity z filled by a worker with productivity θ is denoted $J(z,\theta)$. This value satisfies the following HJB equation:

$$RJ(z,\theta) = z\theta - \theta - \beta z\theta + \sigma(W - J(z,\theta)). \tag{6}$$

The first terms on the right add up to the flow of profit, where $z\theta$ is revenue, θ is salary costs and $\beta z\theta$ are other costs. The remaining term is the separation value, which is the difference between the value of a vacancy, W, and the job, weighted by the separation rate (in the initial steady-state only exogenous separations occur).[7] The steady-state value of a job is then:

$$J^*(z,\theta) = \frac{((1-\beta)z - 1)\theta + \sigma W}{R+\sigma}. \tag{7}$$

The value of a vacancy satisfies

$$RW = -\xi + q \sum_i \int_\theta \int_z \int_b (J(z,\theta) - W) \frac{g(i,b,\theta,u)}{U} P_z(z|\theta) db dz d\theta, \tag{8}$$

where ξ is the flow-cost of posting a vacancy, q is the vacancy filling rate, and $g(i,b,\theta,u)$ is the measure of unemployed workers with discount rate ρ_i, wealth b and skill θ at a point in time. We assume a free-entry condition pushes the value of a vacancy to $W = 0$, and thus the vacancy filling rate at any time is determined by equating the RHS of (8) with zero.

2.1.4 Government:

The government budget constraint in the initial steady-state is

$$G = \mathcal{T} - H, \tag{9}$$

where $\mathcal{T}$ is total tax revenue and H is total spending on unemployment insurance benefits. We show how these are calculated in Appendix A. The government does not borrow to finance spending in steady-state; however, residual expenditure G includes costs to service any existing debt.

2.1.5 Initial Steady-State Equilibrium

We formally define the initial steady-state equilibrium in Appendix B. As is usual in such an equilibrium, all value functions and decisions rules are stationary, the equilibrium distribution of workers over their state variables is stationary, and the government budget constraint is satisfied.

2.2 Pandemic Period

At $t = 0$ a proportion ϕ of jobs are forced to stop producing. We refer to jobs where production is halted as the "affected sector," as opposed to the "unaffected sector." The lockdown continues until the affected sector reopens at $t = t^{open}$. Agents know the end date of the shutdown. The superscript $i \in \{A, UA\}$ denotes affected and unaffected sectors, respectively, for example U_t^A is the unemployment rate in the affected sector. Workers cannot change sectors in the short-run.

[7] More formally, all matches continue to generate positive value outside of the pandemic period, thus no firm would ever choose to lay off their worker. During the pandemic this will change such that endogenous layoffs can occur.

The wage subsidy and/or enhanced unemployment insurance programs begin at $t = 0$, and the government begins issuing debt to finance these programs. Under the wage subsidy, the government pays a proportion $\psi(\theta)$ of the salary of a retained worker in the affected sector, as follows:

$$\psi(\theta) = \begin{cases} \psi_0 & \text{if } \theta \leq \overline{\theta} \\ \psi_0 \times (\overline{\theta}/\theta) & \text{if } \theta > \overline{\theta} \end{cases}. \tag{10}$$

When earnings exceed the ceiling $\overline{\theta}$ the program pays a fixed amount $\psi_0\overline{\theta}$. Enhanced unemployment insurance pays an adjusted benefit $\tilde{h}(\theta)$ to unemployed workers in the affected sector.

During the shutdown period endogenous worker-firm separations may occur. In making layoff decisions firms weigh the cost of maintaining payroll during the pandemic against the benefit of resuming production immediately after the pandemic ends. Workers who are laid off remain temporarily attached to their job, but these attachments dissipate over time. This means that a laid off worker is less likely to be available when production restarts compared to if they remained on payroll. We refer to laid off workers who are still in contact with their firm as "attached unemployed." Such workers may (i) exogenously become conventional "unattached unemployed," (ii) match with a new job and become employed, or (iii) be recalled from layoff.

The value of having a worker on payroll grows faster than the value of layoff over the shutdown period because the time that a firm must pay their idle worker is getting smaller. This has two important consequences for our model. First, some firms may recall attached-unemployed workers before the shutdown ends, meaning endogenous worker recalls are important to allow for. Second, if a worker is laid off from their job, it happens at the moment the shutdown begins, meaning endogenous layoffs only need to be allowed for at the first instant of the pandemic.

Because of the possibility of attached-unemployment, the labor force status possibilities of an affected-sector worker are $s \in \{e, u, a\}$ during the shutdown, where a is attached-unemployment. As will become clear, job productivity z matters for attached-unemployed workers, and thus becomes a state variable for affected sector workers. Given this, the probability measure of such workers is denoted $\tilde{g}_t^A(i, b, \theta, s, z)$, where the dependence of this transitional probability measure on time is captured with a subscript t. The probability measure over the usual state variables can be recovered by $g_t^A(i, b, \theta, s) = \int_z \tilde{g}_t^A(i, b, \theta, s, z)dz$.

2.2.1 Firms

The value of a filled job in the affected sector, $J_t^A(z, \theta)$, now depends directly on time. This value may be substantially different than the initial steady-state job value function, and firms must decide whether to keep their workers or lay them off.

Workers who the firm lays off enter the state of attached unemployment. However, attached-unemployed workers may become conventional (unattached) unemployed workers at an exogenous rate φ. We assume $\varphi > \sigma$ so that firms exogenously lose laid off workers more rapidly than employed workers. In addition, attached-unemployed workers search for new jobs in the same manner as unattached-unemployed workers do. If an attached-unemployed worker finds a job then they leave the firm that laid them off, thus attachments also dissolve endogenously at the job finding rate.

Formally, a firm in the affected sector chooses between keeping their worker on payroll, the value of which is $K_t^A(z,\theta)$, and laying their worker off so that they become attached-unemployed, the value of which is $L_t^A(z,\theta)$. At any point in time the option with the larger value is chosen by the firm, thus $J_t^A(z,\theta) = \max\left\{K_t^A(z,\theta), L_t^A(z,\theta)\right\}$ is the value of a firm in the affected sector during the shutdown. Any worker currently on payroll can be laid off, and any worker currently laid off (and still attached) can be recalled and put back on payroll, depending which option maximizes the value of the firm at that point in time. $K_t^A(z,\theta)$ solves

$$\begin{aligned} RK_t^A(z,\theta) &= -(1-\psi(\theta))\theta + \sigma\big(W_t^A - K_t^A(z,\theta)\big) + \partial_t K_t^A(z,\theta), && \forall t \in [0, t^{open}) \\ K_{t^{open}}^A(z,\theta) &= J^*(z,\theta), && t = t^{open}. \end{aligned} \tag{11}$$

A worker kept on payroll costs the firm the unsubsidized part of their salary, thus the flow value is $-(1-\psi(\theta))\theta$. Exogenous separations may still occur at rate σ, in which case the value becomes that of an affected sector vacancy, W_t^A. The value of maintaining this job increases as time t^{open} approaches, which is captured by $\partial_t K_t^A(z,\theta) > 0$. When the shutdown ends, workers can immediately resume production, which is captured by the terminal condition $K_{t^{open}}^A(z,\theta) = J^*(z,\theta)$.

We assume that firms will only hire workers for jobs that are worth keeping during the shutdown, formally those for which $K_t(z,\theta) \geq 0$. As such, the value of laying off a worker, who then becomes an attached-unemployed, is:

$$\begin{aligned} RL_t^A(z,\theta) =& \left(\varphi + f_t^A \int_{\tilde{z}} P(\tilde{z}|\theta)\mathcal{I}\{K^A(\tilde{z},\theta) \geq 0\}d\tilde{z}\right)(W_t^A - L_t^A(z,\theta)) \\ & + \partial_t L_t^A(z,\theta), && \forall t \in [0, t^{open}) \\ L_{t^{open}}^A(z,\theta) =& J^*(z,\theta), && t = t^{open}. \end{aligned} \tag{12}$$

The flow value of having an attached-unemployed worker consists of two terms on the RHS. The first term reflects the possibility that attachment breaks exogenously, which occurs at rate φ, or endogenously, which occurs at the job finding rate. Because some combinations of worker and job productivity might generate negative value, the job finding rate depends on both the affected sector matching rate f_t^A, and the proportion of matches that have positive value, which is captured by $\int_{\tilde{z}} P(\tilde{z}|\theta)\mathcal{I}\{K(\tilde{z},\theta) \geq 0\}d\tilde{z}$. At the end of the shutdown, any remaining attached-unemployed workers would be recalled, which is captured by the terminal condition $L^A_{t_{open}}(z,\theta) = J^*(z,\theta)$.

Given the assumptions about job posting by firms during the shutdown, the free-entry condition in the affected sector now becomes:

$$\begin{aligned} RW^A_t = &- \xi + q^A_t \sum_i \int_b \int_\theta \int_z (K^A_t(z,\theta) - W^A_t) \frac{g^A_t(i,b,\theta,u) + g^A_t(i,b,\theta,a)}{U^A_t} \times \\ &P_z(z|\theta)\mathcal{I}\{K^A_t(z,\theta) \geq 0\} dz d\theta db. \end{aligned} \tag{13}$$

The expected value of a match depends on the affected-sector measures of unattached- and attached-unemployed workers, $g^A_t(i,b,\theta,u)$ and $g^A_t(i,b,\theta,a)$, respectively, as well as the affected-sector unemployment rate U^A_t. Firms are not allowed to lay off new hires during the pandemic in order to ensure that all layoffs occur at $t = 0$.

Firm decisions in the unaffected sector continue as in the initial steady state, or in other words $J^{UA}_t(z,\theta) = J^*(z,\theta)$, because production is allowed to continue there as usual.

2.2.2 Workers

The value function of an affected sector worker, $V^A_t(i,b,\theta,s,z)$, depends on their job productivity, as discussed above. However, this state-variable only applies to attached-unemployed workers, and so is repressed for other workers.

First consider an unattached-unemployed worker in the affected sector, who solves the following problem during the shutdown:

$$\begin{aligned} \rho_i V^A_t(i,b,\theta,u) = &\max_c v(c) + \partial_b V^A_t(i,b,\theta,u)\dot{b} \\ &+ f^A_t \int_z P_z(z|\theta)\mathcal{I}\{K^A_t(z,\theta) \geq 0\} dz \big(V^A_t(i,b,\theta,e) - V^A_t(i,b,\theta,u)\big) \\ &+ \sum_{j \neq i} \pi_\rho \big(V^A_t(j,b,\theta,u) - V^A_t(i,b,\theta,u)\big) + \partial_t V^A_t(i,b,\theta,u) \\ \text{s.t. } \; &\dot{b} = \tilde{h}(\theta) + br(b) - c, \;\; b \geq \underline{b}. \end{aligned} \tag{14}$$

Affected sector workers match with potential employers at rate f^A_t, but some matches have negative value and so the job finding rate is smaller, depending on the proportion of such matches for which $K_t(z,\theta) \geq 0$. When a job is found the value function jumps to $V^A_t(i,b,\theta,e)$. Notice that the unemployment benefit is now $\tilde{h}(\theta)$.

Attached-unemployed workers solve a similar problem as their unattached counterparts, with two primary differences. First, there will be a term in the attached-worker HJB equation that reflects the possibility of losing attachment and becoming an unattached-unemployed worker. Second, attached-unemployed workers anticipate that they may be recalled at some future date. Given the assumptions of our model, workers know with certainty when their firm would recall them. This provides a boundary condition that their value function must satisfy, as they will have the same value function as an employed worker at that time, assuming attachment does not dissolve beforehand. A simple way to express this boundary condition is to impose that the value of an attached worker be equal to that of an equivalent employed worker whenever $K_t^A(z,\theta) \geq L_t^A(z,\theta)$, or in other words whenever the firm will prefer to have the worker on payroll. Formally, attached-unemployed workers solve the following problem:

$$\begin{aligned}
\rho_i V_t^A(i,b,\theta,a,z) = \max_c\, & v(c) + \partial_b V_t^A(i,b,\theta,a,z)\dot{b} \\
& + f_t^A \int_{\tilde{z}} P(\tilde{z}|\theta)\mathcal{I}\{K_t(\tilde{z},\theta) \geq 0\} d\tilde{z} \cdot \big(V_t^A(i,b,\theta,e) - V_t^A(i,b,\theta,a,z)\big) \\
& + \varphi\big(V_t^A(i,b,\theta,u) - V_t^A(i,b,\theta,a,z)\big) \\
& + \sum_{j\neq i} \pi_\rho\big(V_t^A(j,b,\theta,a,z) - V_t^A(i,b,\theta,a,z)\big) + \partial_t V_t^A(i,b,\theta,a,z) \\
\text{s.t. } & \dot{b} = \tilde{h}(\theta) + br(b) - c, \;\; b \geq \underline{b}, \\
\& \;\; & V_t^A(i,b,\theta,a,z) = V_t^A(i,b,\theta,e)\, \forall t \mid \big(K_t^A(z,\theta) \geq L_t^A(z,\theta)\big)
\end{aligned} \tag{15}$$

The second line captures the possibility of finding a new job before recall occurs, while the third reflects the possibility of exogenous loss of attachment to the job. The boundary condition is specified in the last line.

Employed workers in the affected sector solve a similar problem to that of the initial steady-state, except that their value function depends on time for several reasons. Such workers solve the following problem:

$$\begin{aligned}
\rho_i V_t^A(i,b,\theta,e) = \max_c\, & v(c) + \partial_b V_t^A(i,b,\theta,e)\dot{b} \\
& + \sigma\big(V_t^A(i,b,\theta,u) - V_t^A(i,b,\theta,e)\big) \\
& + \sum_{j\neq i} \pi_\rho\big(V_t^A(j,b,\theta,e) - V_t^A(i,b,\theta,e)\big) + \partial_t V_t^A(i,b,\theta,e) \\
\text{s.t. } & \dot{b} = \theta - T(\theta) + br(b) - c, \;\; b \geq \underline{b}.
\end{aligned} \tag{16}$$

Future changes in government policy cause the value function to change over time, captured by $\partial_t V_t^A(i,b,\theta,e)$, and the continuation value in the event of job loss also varies with time.

During the shutdown unaffected workers continue to solve similar problems as in (3) and (4) with the same job finding rate as in the steady state. The main difference is that unaffected workers foresee an increase in future taxes, and respond accordingly by altering their consumption decisions. As such, their value functions depend directly on time, and are now denoted $V_t^{UA}(i,b,\theta,s)$, and in general $\partial_t V_t^{UA} \neq 0$. This implies an additional term in the HJB equations of unaffected workers, for example the HJB equation of an unemployed worker in the unaffected sector is

$$\begin{aligned} \rho_i V_t^{UA}(i,b,\theta,u) = & \max_c v(c) + \partial_b V_t^{UA}(i,b,\theta,e)\dot{b} + f\big(V_t^{UA}(i,b,\theta,e) - V_t^{UA}(i,b,\theta,u)\big) \\ & + \sum_{j\neq i} \pi_\rho \big(V_t^{UA}(j,b,\theta,u) - V_t^{UA}(i,b,\theta,u)\big) + \partial_t V_t^{UA}(i,b,\theta,u) \\ \text{s.t. } & \dot{b} = h(\theta) + r(b)b - c, \quad b \geq \underline{b}, \end{aligned} \tag{17}$$

and similar for unaffected employed workers.

2.2.3 Government

During the economic shutdown the government runs the wage subsidy program, offers expanded unemployment benefits, and borrows in order to balance their budget. The accumulated debt associated with the pandemic at time t is denoted B_t. The pandemic-era cost of unemployment benefits, H_t, will be higher than in the initial steady-state due to higher unemployment rates and (possibly) more generous benefits. The cost of the wage subsidy program is captured by an additional expenditure term Q_t. See Appendix A for more details on how these terms are calculated. With these pandemic features, the federal budget constraint at a point in time is

$$\dot{B}_t = rB_t + Q_t + H_t + G - \mathcal{T}_t. \tag{18}$$

Note that tax revenue $\mathcal{T}_t$ will also differ from the initial steady-state because of lower employment rates. Residual expenditure continues at the initial steady-state level G.

2.3 Recovery and Post-Recovery Periods

The recovery period begins at $t = t^{open}$. To the extent that unemployment is still above the steady-state level at $t = t^{open}$, the labor market frictions of the model imply a period of transition back to a stationary labor market. The recovery period ends at time $t = t^{rec}$ when the unemployment rate returns to the steady-state level.

The wage subsidy and/or enhanced unemployment benefit programs end at t^{open}; however, the government still needs to borrow in order to meet debt servicing costs because tax rates have not yet been increased. Instead, we assume that the government waits until the post-recovery period has begun at $t = t^{rec}$ before raising taxes. At this point the government stops rolling over debt, and taxes increase by enough to pay for the programs introduced during the shutdown. A period of transitional dynamics continues after $t = t^{rec}$, despite the fact that the labor market is in a stationary-state, because the household wealth distribution still needs to adjust towards the final steady-state.

2.3.1 Government

During the recovery period the government budget constraint is as in (18), but with $Q_t = 0$.

In the post-recovery period, from t^{rec} onwards, the government is ready to begin paying the debt accumulated during the pandemic. At this point a quantity of debt $B_{t^{rec}}$ has accumulated. We assume that the government converts this debt to a perpetuity, and thus will make payments $rB_{t^{rec}}$ for the rest of time. To accommodate this cost, tax rates are revised such that $\tilde{T}(\theta)$ is the new tax schedule. In the calibrated model the initial average tax rate will take the functional form $T(\theta)/\theta = 1-\lambda(\theta/E[\theta])^{-\gamma}$ as in Heathcote, Storesletten, and Violante (2017), where $E[\theta]$ is cross-sectional mean earnings in steady-state. The parameter λ will adjust to balance the budget, leaving the progressivity parameter γ unchanged. Under this new tax schedule government revenue will become $\tilde{\mathcal{T}}$. The post-recovery government budget constraint is then

$$rB_{t^{rec}} + G = \tilde{\mathcal{T}} - H. \tag{19}$$

This equation determines the post-recovery tax reform, given the debt accumulated under the policies that existed during the pandemic.

2.3.2 Workers

In the recovery period all jobs have positive value, thus any match will again result in a job being created. Furthermore, all attached-unemployed workers will be recalled. The HJB equations of unaffected workers continue to be as during the pandemic, for example (17) is still the problem of an unaffected unemployed worker after the lockdown ends but before the labor market has recovered ($t^{open} \leq t < t^{rec}$).

The HJB equations of affected sector workers also continue to vary with time until the end of the recovery period. The job finding rate of the workers, f_t^A, continues to evolve as the labor market recovers, and so an unemployed affected sector worker has the following problem from t^{open} to t^{rec}:

$$\begin{aligned}
\rho_i V_t^A(i,b,\theta,u) = \max_c\, & v(c) + \partial_b V_t^A(i,b,\theta,u)\dot{b} \\
& + f_t^A\big(V_t^A(i,b,\theta,e) - V_t^A(i,b,\theta,u)\big) \\
& + \sum_{j\neq i} \pi_\rho\big(V_t^A(j,b,\theta,u) - V_t^A(i,b,\theta,u)\big) + \partial_t V_t^A(i,b,\theta,u) \\
\text{s.t. }\; \dot{b} = h(\theta) + & br(b) - c, \quad b \geq \underline{b}.
\end{aligned} \tag{20}$$

Unlike during the lockdown, all matches result in hires, so the job finding rate is again the matching rate. Also, the unemployment benefit is back to the baseline schedule $h(\theta)$. The HJB equation of an employed worker from t^{open} to t^{rec} is the same is in (16)

When time t^{rec} arrives the tax schedule changes from $T(\theta)$ to $\tilde{T}(\theta)$, and there will be no further changes to the worker problems thereafter. As such, worker value functions will be the same as the final steady-state value functions, which we denote $V^f(i, b, \theta, s)$. That is, in the post-recovery transition period workers will solve the same problems as in the initial-steady state (specifically 3 and 4), without any time dependence of the value functions. The only caveat is that the change in the tax function. Although the value functions will be stationary, the distribution $g_t(i, b, \theta, s)$ will continue to evolve until the final steady-state distribution is attained.

2.3.3 Firms

The affected sector is allowed to resume production, thus the job posting decisions of firms in both sectors are governed by the same equations as in the initial steady-state. However, the affected sector vacancy filling rate, q_t^A, will vary through the recovery period as the affected sector unemployment rate falls. In the post-recovery period (after $t = t^{rec}$) the unemployment rates of both sectors will be at their long-run levels, and the overall labor market will be in a stationary state.

2.4 Equilibrium

We formally define a recursive competitive equilibrium for the pandemic and post-pandemic periods in Appendix B. Although there are many moving parts in the model, and hence many unique equilibrium objects to pin down, there is nothing non-standard about our equilibrium concept. Agents are rational price-taking optimizers and the government budget constraint must be satisfied, as has just been described.

3 Calibration

Government: The government taxes income according to the HSV tax function, as in Heathcote, Storesletten, and Violante (2017). In particular, the average tax rate of a type-θ worker takes the form $T(\theta)/\theta = \tau(\theta) = 1 - \lambda(\theta/E[\theta])^{-\gamma}$, where γ governs the progressivity of the tax system, λ dictates the average level of taxation in the economy, and $E[\theta]$ is mean earnings in the steady-state economy. We set $\lambda = 0.902$ and $\gamma = 0.036$ according to the estimates by Guner, Kaygusuz, and Ventura (2014). For unemployment benefits we set $h(\theta) = 0.4\theta(1 - \tau(\theta))$, which implies a replacement rate of 40% of net earnings.[8]

[8] Further robustness checks regarding these calibration details will be provided in section 4.6.

In the post-recovery period the government must balance a budget that now includes perpetuity payments $rB_{t^{rec}}$ as in equation (19). To achieve this, we assume that the government raises the average tax level in the economy by adjusting λ to $\tilde{\lambda}$, and thus the post-recovery average tax function is $\tilde{\tau}(\theta) = 1 - \tilde{\lambda}(\theta/E[\theta])^{-\gamma}$.

Labour Market Parameters: The labor market parameters include those of the matching function $M(U, \nu)$, the exogenous separation rate σ, and the affected proportion of the economy ϕ. We assume that the matching function is Cobb-Douglas of the form $M(U, \nu) = \chi U^{\eta} \nu^{1-\eta}$, where matching efficiency χ and elasticity η are the parameters to be calibrated. The attachment-dissolution rate, φ, governs the measure of unemployed workers that get recalled when the shutdown ends.

We follow Shimer (2005) closely in choosing the basic parameter values. We set the $\eta = 0.72$, $\sigma = 0.008$, and set χ and ξ such that the steady-state job finding rate and market tightness are 0.104 and 1, respectively. In particular, targeting the market tightness of 1 implies that $f_t = \chi$. Conditional on the calibration of μ_z, the fixed cost of posting a vacancy is then:

$$\xi = \chi \int_{\theta} \int_{z} \frac{((1-\beta)z - 1)\theta}{R + \sigma} P(z, \theta) dz d\theta, \tag{21}$$

Where $P(z, \theta)$ is the joint density of z and θ. This equation is equivalent to the initial steady-state free-entry condition (8), when the market tightness is 1, and $J_t(z, \theta)$ replaced by its value in the initial steady-state. The calibration implies that on average employment lasts for about 2.5 years, and average duration of unemployment is about 2.5 months.

The parameter that determines the rate at which attached-unemployed workers become unattached is set based on CPS data from April to September of 2020. In particular, for each month we compute the proportion of individuals who report a transition from temporary unemployment that month to any other form of unemployment in the following month. We interpret this as a transition from attached to unattached unemployment, which provides a measure of φ for that month. We average over the six months that we utilize, weighting by the number of new temporarily unemployed workers that month. We find that the weekly rate at which attached-unemployed workers transition to conventional unemployment is $\varphi = 0.045$. It is worth noting that this rate is about half as large as would be consistent with the data reported in Fujita and Moscarini (2017), which shows that the recall rate is much higher after a pandemic lockdown than in normal times. This is consistent with the quick recovery of the U.S. unemployment rate during the pandemic, relative to other recessions as shown in Hall and Kudlyak (2020).

The last labor market parameter is the affected proportion of the economy ϕ, which we calibrate using data on U.S. firms in January 2021, documented by Damodaran (2021). We calculated the number of firms that have a negative profit margin during this period, and divide it by the total number of firms in the sample. The result is $\phi = 0.23$, which implies that 23% of the firms are in the affected sector.

Household Parameters: Household parameters include the flow utility risk aversion parameter ς, the three values of the discount rate ρ, the discounting type transition rate π_ρ, the borrowing limit $\underline{b}$, the interest rates r and R, and the parameters of the distribution of productivity θ. To begin with, we specify the earnings distribution directly from data. With our earnings function $y(\theta) = \theta$, we can back out the distribution of θ directly from weekly earnings observed in data. In the model we approximate the distribution of weekly earnings with a log-normal distribution, implying that $\log(\theta) \sim \mathcal{N}(\mu_\theta, \sigma_\theta^2)$. We use the 2019 CPS March Supplement to estimate μ_θ and σ_θ^2. Details can be found in Appendix D. We find that $\mu_\theta = 6.755$ and $\sigma_\theta^2 = 0.8027$. These estimates imply that mean weekly earnings are about \$1127.

Three of the remaining parameters are specified by assumption. The return on saving r is set to the equivalent of a 4% annual rate, the coefficient of relative risk aversion is set to $\varsigma = 2$, and the rate at which discounting transitions occur is set to $\pi_\rho = 1/4160$. This value of π_ρ implies a jump once every 40 years on average[9], leading to a generational interpretation of the discount rate value.

The remaining parameters are internally calibrated so that the distribution of wealth in the model matches important features of the distribution in the data. We use the 2019 Survey of Consumer Finances for this purpose. To calibrate the borrowing limit and borrowing interest rate, we target features of the negative part of the wealth distribution. In particular, we seek to replicate that 10.48% of households in the data had negative net worth in 2019, and that the average value of unsecured debt among those borrowers was \$8,478.[10] The logic of these choices is that the borrowing wedge $R - r$ will regulate how many people want to borrow, while the borrowing limit will restrict how large the biggest loans are, and therefore average debt. The calibrated borrowing limit is $\underline{b}$ =-\$24,400, and the borrowing interest rate R is the equivalent of 5.25% annually. These parameters allow the model to explain the data well, generating 10.56% of workers with negative net worth, and an average debt level of \$8,660.

For the discount rate process we reduce the number of parameters by making the possible discount rate values be $\rho_i \in \{\bar{\rho} - \Delta_\rho, \bar{\rho}, \bar{\rho} + \Delta_\rho\}$, so that $\bar{\rho}$ and Δ_ρ are the two parameters to be calibrated. Because lower wealth individuals are the most affected by income fluctuations, we specifically target features of the bottom half of the wealth distribution to calibrate $\bar{\rho}$ and Δ_ρ, in particular the levels of wealth at the 20th and 50th percentiles of the wealth distribution, which are \$6,370 and \$121,760 in the data, respectively. These parameters also have implications for the top

[9]To see this, notice that a household of type ρ_1 will transit to type ρ_2 and ρ_3 at the same Poisson rate π_ρ, therefore the average duration of staying being type ρ_1 is $\frac{1}{\pi_\rho + \pi_\rho}$. We set the average duration to 2080 weeks and solve for π_ρ

[10]Unsecured debt is measured as the value of debt for goods and services, which is broader than credit card debt.

of the wealth distribution, particularly Δ_ρ, and we return to this when assessing the performance of the model below. The calibrated parameter values are $\bar{\rho} = 0.0444$ and $\Delta_\rho = 0.020$, which generate \$6,881 of wealth at the 20th percentile and \$113,177 at the 50th percentile in the model.

Firm Parameters: The behavior of firms depends on the labor market parameters specified above, along with the discount rate R, conditional distribution of job productivity $P_z(z|\theta)$ and the parameter determining non-labor operating costs β. However, given that firm profit can be re-written $(1-\beta)z\theta - \theta$, it is not necessary to separately calibrate β and z. Rather, we calibrate the distribution of a compound variable $z^* = (1-\beta)z$, with $z^* > 1$ assumed. The new variable is substituted into the problems described above, and thus we must parameterize the distribution $P_{z^*}(z^*|\theta)$, rather than $P_z(z|\theta)$. We proceed to calibrate the model by parameterizing the distribution of a transformation of the new variable $Z \equiv 1 - \dfrac{1}{z^*}$. Z is useful because it is related to profit margins. In particular, let $p_1 = (z\theta - \theta - \beta z\theta)/z\theta$ be the net-margin of a firm, and $p_2 = (z\theta - \theta)/z\theta$ be the profit margin before other expenses are subtracted. We can then derive Z from these observable profit margins according to $Z = p_1/(p_1 - p_2 + 1)$. We use data from January 2020 provided by Damodaran (2021), using net margin to measure p_1, and earnings before interest, taxes, depreciation, amortization, and general and administrative expenses, relative to sales, (EBITDAG&A/sales) to measure p_2.

Next, we proceed by assuming that worker productivity and firm margin (Z) are jointly log normally distributed in steady-state[11], which implies that the conditional distribution of Z is log-normal as follows:

$$\ln(Z)|\ln(\theta) = x \sim \mathcal{N}\left(\mu_z + \frac{\sigma_z^2}{\sigma_\theta^2}\alpha(x - \mu_\theta), (1-\rho^2)\sigma_z^2\right). \tag{23}$$

μ_z and σ_z^2 are the mean and variance of $\ln(Z)$, and α is the correlation coefficient between $\ln(Z)$ and $\ln(\theta)$. The conditional distribution $P_{z^*}(z^*|\theta)$, which enters firm's value functions, is then computed from this distribution.[12] Choi, Haque, Lee *et al.* (2013) use the US Census data to estimate that the correlation between labor productivity and gross profit margin per worker is approximately 0.1, and we accordingly set $\alpha = 0.1$. Our assumptions also imply that Z itself follows a log-normal distribution, and the January 2020 data on profit margins described above imply $\mu_z = -2.3909$ and $\sigma_z = 0.7368$.

[11]Specifically, Z and θ follow a joint log-normal distribution where

$$\begin{bmatrix}\ln(Z)\\ \ln(\theta)\end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix}\mu_z\\ \mu_\theta\end{bmatrix}, \begin{bmatrix}\sigma_z^2 & \alpha\sigma_z\sigma_\theta\\ \alpha\sigma_z\sigma_\theta & \sigma_\theta^2\end{bmatrix}\right) \tag{22}$$

The distribution of z can be numerically backed out from the distribution of Z

[12]In practice we discretize $\ln(Z)$ and $\ln(\theta)$, and each $(\ln(Z), \ln(\theta))$ pair is associated with a unique (z^*, θ) pair that occurs with the same probability.

Table 1: Calibration Summary

A. Externally Specified or Normalized			Source		
Matching efficiency	χ	0.104	Shimer (2005)		
Matching elasticity	η	0.72	Shimer (2005)		
Job separation rate	σ	0.008	Shimer (2005)		
Cost of posting vacancy*	ξ	1.77	Normalization (eq. 21)		
CRRA coefficient	ζ	2	Heathcote, Storesletten, and Violante (2014)		
Average tax parameter	λ	0.902	Guner, Kaygusuz, and Ventura (2014)		
Tax progressivity	γ	0.036	Guner, Kaygusuz, and Ventura (2014)		
Real rate of return**	r	0.04	Assumption		
Discount rate jump rate**	π_ρ	0.0125	40 year average duration		
wage-profit marg. correlation	α	0.1	Choi, Haque, Lee *et al.* (2013)		
B. Estimated Outside the Model			**Data Source**		
Attachment-dissolution rate	φ	0.045	a-u transition rate (CPS Monthly Apr-Sept 2020)		
Mean of log wage	μ_θ	6.755	See Appendix D (CPS March 2019)		
Variance of log wage	σ_θ^2	0.803	See Appendix D (CPS March 2019)		
Size of affected sector	ϕ	0.23	Prop. of firms with negative net margin 2020 (Damodaran, 2021)		
Mean of log profit margin	μ_z	-2.3909	Dist. of profit margins Damodaran (2021)		
Variance of log profit margin	σ_z^2	0.7368	Dist. of profit margins Damodaran (2021)		
C. Internally Calibrated (jointly determined)			**Moment**	**Data**	**Model**
Borrowing limit	$\underline{b}$	-\$24,400	Prop. with $b < 0$ (SCF 2019)	10.48%	10.56%
Borrowing interest rate**	R	0.0525	$E[b\|b < 0]$ (SCF 2019)	\$8,478	\$8,660
Mean discount rate**	$\bar{\rho}$	0.0444	20th pctl. of b (SCF 2019)	\$6,370	\$6,881
Discount rate variation**	Δ_ρ	0.020	50th pctl. of b (SCF 2019)	\$121,760	\$113,177

* Expressed relative to the mean weekly wage, i.e. the true value is $\xi = 1.77 \times E[\theta]$
** Expressed as an annualized rate, as opposed to weekly.

Shutdown parameters: In the baseline exercise, we shutdown the affected sector for 12 weeks. In all exercises, we assume that the recovery takes at most 100 weeks after the shutdown begins. In all cases, 100 weeks is enough for the unemployment rate to fully recover. As such, we set $t^{open} = 12$ and $t^{rec} = 100$. Varying the assumed length of the lockdown is an important experiment we consider below.

3.1 Assessing Model Performance

We assess the performance of our model in explaining two untargeted features of the data. The first is the nature of the time-path of unemployment during a pandemic. To this end, Figure 1 displays two versions of the pandemic unemployment rate for both the data and model. In the LHS we plot the official monthly unemployment rate alongside the unemployment rate path in our model, assuming a 12 week shutdown duration. In the RHS we plot an alternative version of the data, where the unemployment rate is adjusted for changes in the labor force participation rate during the pandemic. Specifically, in the adjusted version of the data we take the difference between the total labor force in February 2020 and that in each following month, and add this to the number of officially unemployed workers. For both plots the peak unemployment rate in April 2020 is aligned with the first week of the pandemic in the model. While the model overshoots the unemployment jump relative to the official unemployment rate, it captures the jump in the adjusted unemployment rate almost exactly. Given that workers cannot leave the labor force in our model, the right hand

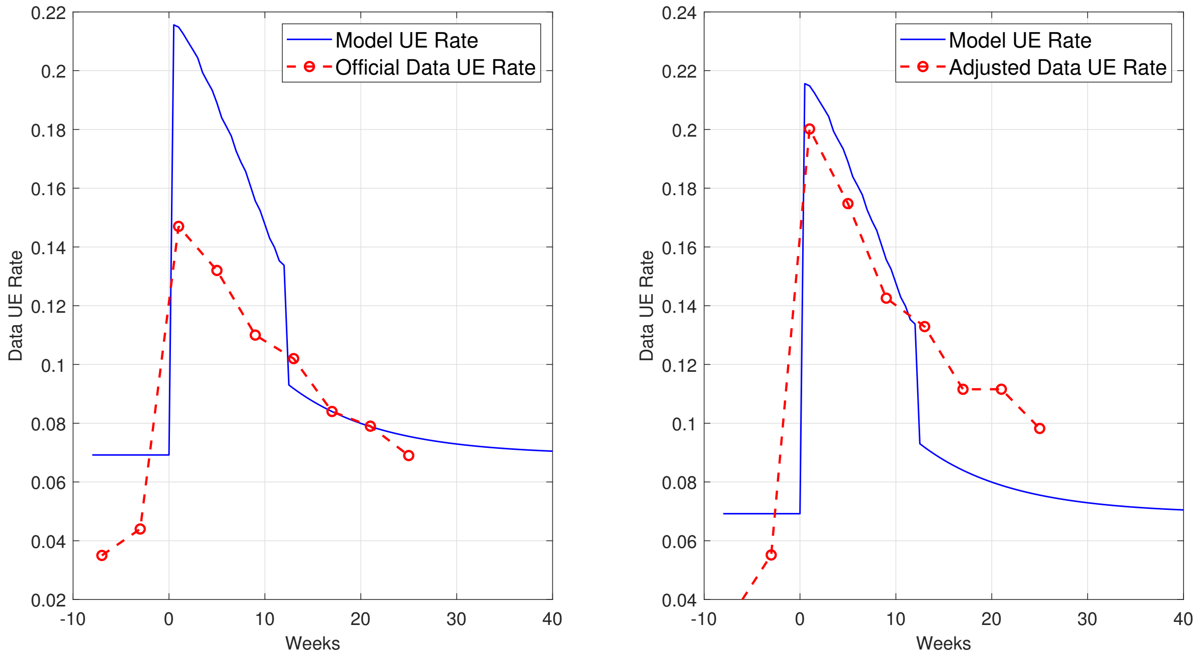

Figure 1: Pandemic unemployment rates in the model and data. The left hand plot includes data on the official monthly unemployment rate, with the peak corresponding to April 2020. The right hand plot includes discouraged workers in the number of unemployed workers, as described in the text. With discouraged workers accounted for, the size of the jump in the unemployment rate as the pandemic begins is very similar in the model and data.

side can be viewed as the better comparison. As such, Figure 1 shows that our calibration of the pandemic parameters and the distribution of job productivities generates a sensible reaction of the unemployment rate.

The second untargeted data feature we consider is the upper half of the wealth distribution. Table 2 present points along the Lorenz curves of net worth in the 2019 Survey of Consumer Finances and our benchmark model. The bottom half of the wealth distribution in replicated quite well because these are calibration targets. A stronger test of the model is how well it can replicate the untargeted top half of the wealth distribution, which requires the model to generate a thick right tail. The fit of the model to these features is imperfect, but overall we view it as reasonably good. There is somewhat more wealth held by those in the top quintile in the data than in the model. Nonetheless, the simple inclusion of discount rate heterogeneity generates saving behavior where more than 80% of wealth is held in the top quintile. Table 2 also presents more details about the very top of the wealth distribution. From this we see that our model actually does quite well right up to the 99th percentile, and that where it misses the data is among the very top 1% wealthiest households. One can argue that capturing exactly how many billions of dollars these top households own is somewhat irrelevant for our exercise because those levels of wealth imply

Table 2: Distribution of Wealth (% owned by each group)

Economy	Quintile					Top Percentiles	
	First	Second	Third	Fourth	Fifth	90-99th	99-100th
U.S. 2019	-0.52	0.79	3.35	8.96	87.41	40.90	37.19
Benchmark	-0.14	1.11	2.03	14.07	80.20	42.16	21.01

massive insurance against a loss of employment in any case.

4 Counterfactual Policy Analysis

We use counterfactual analysis to address several policy related questions. These include (i) Is a pandemic-era wage subsidy welfare-improving? (ii) What is the optimal wage subsidy schedule? (iii) Under what conditions is the welfare gain from a wage subsidy larger than the welfare gain from a temporary increase in unemployment benefits? (iv) Who gains the most from a wage subsidy? We work through these questions in turn.

4.1 Optimal Wage Subsidy Policy

We seek an optimal combination of wage subsidy parameters ψ_0 and $\overline{\theta}$ that maximize social welfare, taking into account the entire transition path of the economy starting at $t = 0$. To put this formally, let $\mathcal{W}(\psi_0, \overline{\theta})$ be social welfare measured by ex-ante utility, specifically:

$$\begin{aligned}\mathcal{W}(\psi_0, \overline{\theta}) =& \phi \int_b \int_\theta \sum_{i=\{1,2,3\}} \left(\sum_{s=\{e,u\}} V_0^A(i,b,\theta,s) g_0^A(i,b,\theta,s) + \int_z V_0^A(i,b,\theta,s,z) \tilde{g}_0^A(i,b,\theta,s,z) dz \right) db d\theta \\ &+ (1-\phi) \int_b \int_\theta \left(\sum_{i=\{1,2,3\}} \sum_{s=\{e,u\}} V_0^{UA}(i,b,\theta,s) g_0^{UA}(i,b,\theta,s) \right) db d\theta. \end{aligned} \tag{24}$$

Note that the components of $\mathcal{W}(\psi_0, \overline{\theta})$, namely V_0^A, V_0^{UA}, g_0^A, $\tilde{g}_0^A$ and g_0^{UA}, as well as the policy parameters ψ_0 and $\overline{\theta}$, are all part of the equilibrium defined in Appendix B. The planning problem can then be described as choosing ψ_0 and $\overline{\theta}$ to maximize $\mathcal{W}(\psi_0, \overline{\theta})$, subject to the constraint that the choice variables are part of an equilibrium as defined.

In Figure 2 we graph $\mathcal{W}(\psi_0, \overline{\theta})$, transformed into consumption equivalent variation relative to the 'no subsidy' case.[13] Any wage subsidy within the range that we simulate improves welfare compared to no subsidy. Furthermore, welfare is concave in the both policy parameters, resulting

[13]In our model the consumption equivalent is simple $\omega = (W(\psi_0^*, \overline{\theta}^*)/W(0,0))^{\frac{1}{1-\varsigma}} - 1$, where $(\psi_0^*, \overline{\theta}^*)$ is the optimal policy and $(0,0)$ is the no subsidy case.

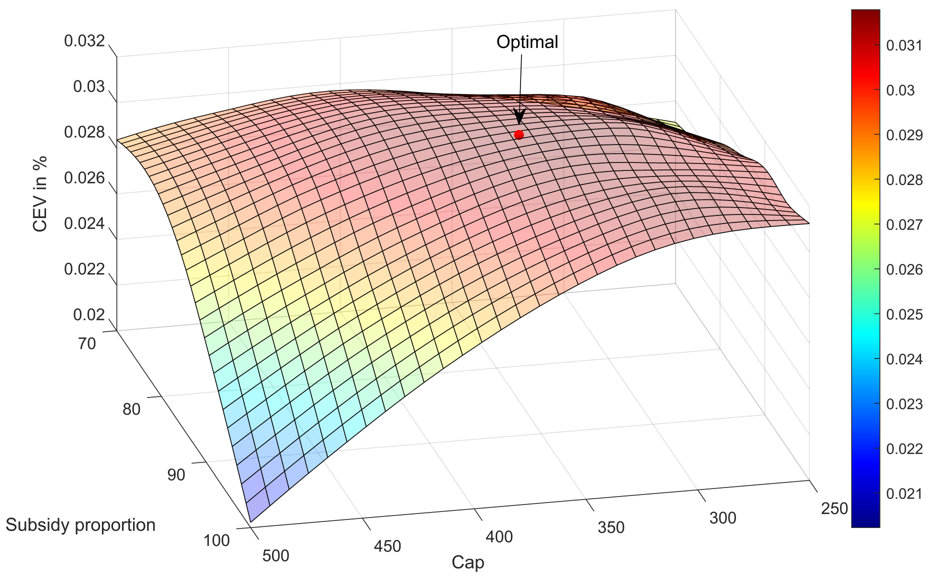

Figure 2: Consumption Equivalent Variation of various $(\psi_0, \overline{\theta})$ combinations compared to the "no subsidy" scheme. The marginal subsidy rate, ψ_0, is in percentage units. The ceiling, $\bar{\theta}$, is in dollars.

in an interior solution for the optimal policy. Specifically, the optimal wage subsidy scheme is $\psi_0^* = .85$ with a ceiling $\overline{\theta}^* = \$350$. This policy implies that many matches in which workers earn less than \$350/week will be kept during the lockdown, regardless of the productivity of their job, and some higher earning workers will also be retained by their employers. On the other hand, about half of the workers whose salaries are subsidized at $t = 0$ would have been kept on payroll anyway. The average weekly cost of such redundant subsidization works out to about 0.063% of steady-state weekly GDP, which limits the scope of the optimal policy.

4.2 UI policy with a ceiling

Instead of offering a wage subsidy, government could increase unemployment insurance benefits in the affected sector (we call this the UI policy). We consider a UI policy that temporarily increases the replacement rate to 100% up to a ceiling, which replicates the full salary received by workers under a wage subsidy.[14] Formally, we solve the model with the following unemployment benefit function:

$$\tilde{h}_t(\theta) = 0.4 \times \theta + \min(0.6 \times \theta, \bar{h}),\ 0 \leq t < t^{open}. \tag{25}$$

[14] One may wonder why we do not allow the planner to choose the replacement rate. If we allowed that the planner would use the unemployment benefit as a redistributive tool because poorer lower skilled workers tend to get laid off. The result would be a replacement rate exceeding 1.

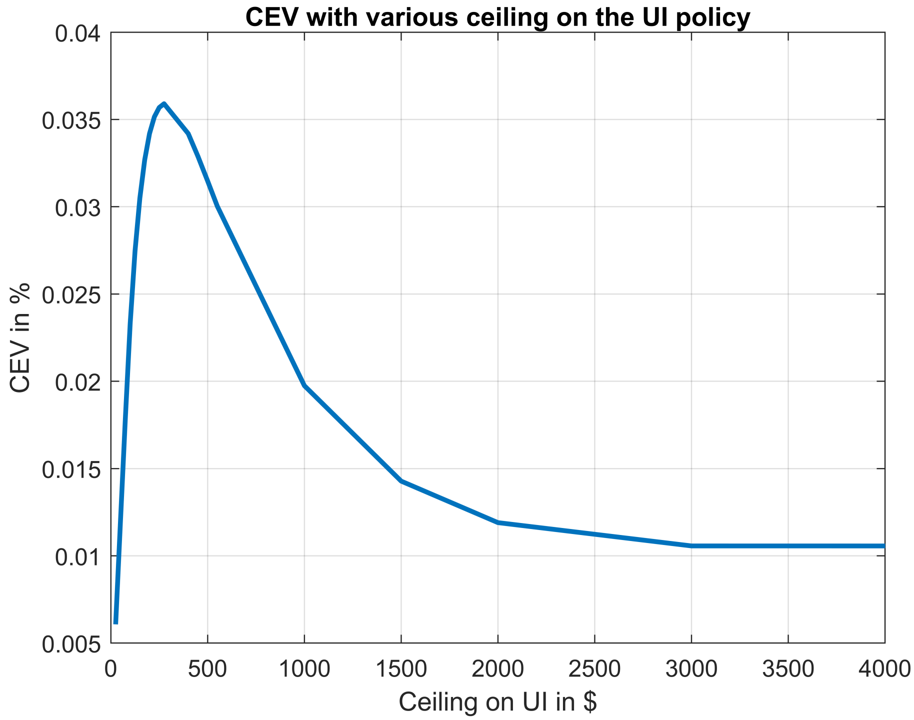

Figure 3: Consumption Equivalent Variation of the UI policy with various ceilings compared to the laissez-faire case

The first term is the benchmark 40% replacement, and the second term represents replacing the remaining 60% of salary up to the ceiling $\bar{h}$. We compute the CEV associated with various values of $\bar{h}$, and present the result in the Figure 3. The CEV is positive for all ceiling values plotted, and there is a clear optimal ceiling around \$275 per week. This is equivalent to fully replacing the income of unemployed workers whose weekly earnings were \$275/week or less, and sending a top-up payment to those who previously earned more.

4.3 Wage subsidy versus increased unemployment benefits

The distribution of firm profit margins is a key factor determining the measure of workers kept on payroll versus laid off. When profit margin is higher, the relative value of keeping an affected job increases, which results in fewer endogenous layoffs and a lesser role for a subsidy. Figure 4 plots the welfare gains from optimal wage subsidy and UI policies over a range of mean profit margins. When average profit margin exceeds 12.25% the UI policy dominates, and vice versa. In our benchmark economy the mean profit margin is 12%, so the wage subsidy slightly dominates.

Shutdown duration also influences the value of keeping a job. In the absence of the subsidy, longer shutdown duration presents a longer stream of wage payments that firms have to make to

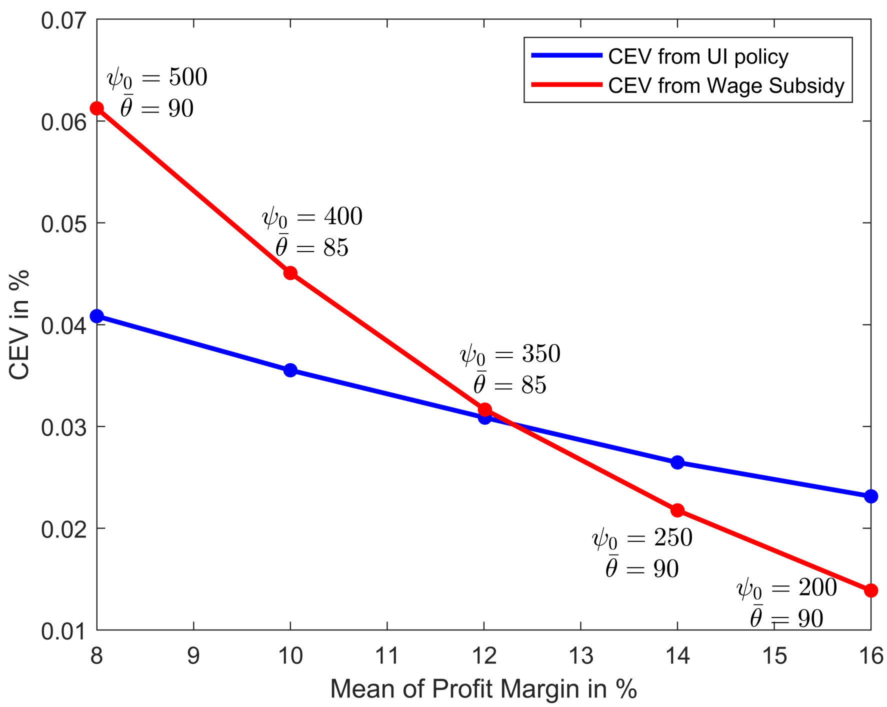

Figure 4: Consumption Equivalent Variation of the optimal wage subsidy scheme compared to the UI policy for various means of profit margin. The subsidy rate, ψ_0 is in percentage. The ceiling, $\bar{\theta}$, is in \$ term

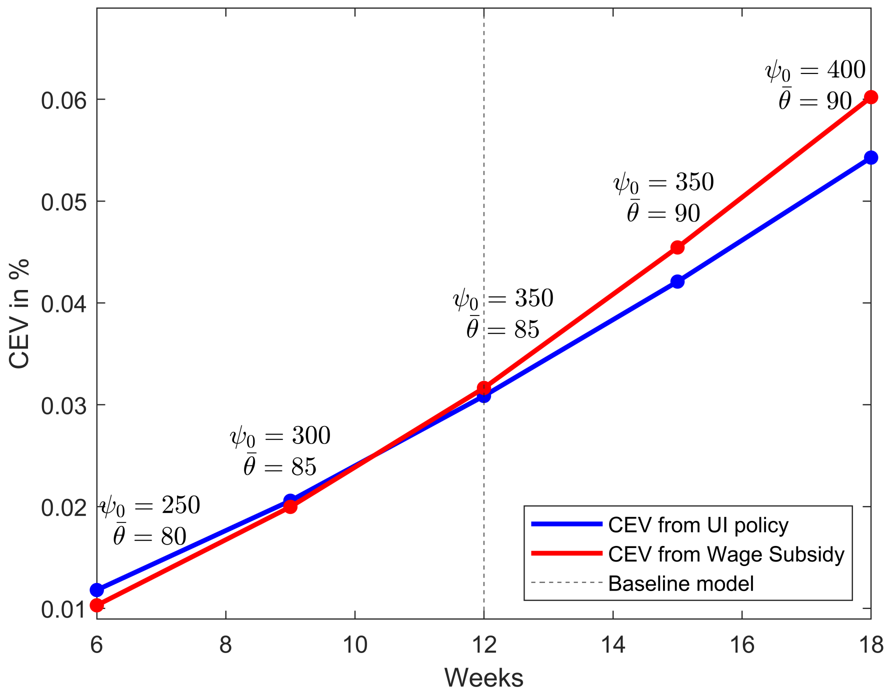

Figure 5: Consumption Equivalent Variation of the optimal wage subsidy scheme compared to the UI policy for length of shutdown duration

keep workers employed, whereas the future value of the job remains the same. As such, the present value of keeping a worker on payroll will be negatively related to shutdown duration. Figure 5 displays the relative welfare gains of the two types of policy over different shutdown lengths. For any shutdown longer than 11 weeks the wage subsidy dominates, and vice-versa.

We also experimented with varying the size of the affected sector, ϕ. While the value of both policies increases with sector size, their ranking is robust to changing this parameter.

4.4 Jointly optimal wage subsidy and UI enhancement

We now solve for an optimal mixed policy that combines the capped UI policy with the wage subsidy. In particular, we find the set of parameters $(\psi_0, \bar{\theta}, \bar{h})$ that maximize the welfare function as in (24). The results are illustrated in Figure 6, which plots the welfare gains from the joint policy across several values of the UI ceiling, along with the welfare gains from the respective UI policies absent the wage subsidy. The optimal UI ceiling is again about \$275, and the optimal subsidy given this ceiling is 90% capped at \$200/week. Thus, although the optimal UI policy is the same as when specified independently, the optimal wage subsidy is different in two ways. First, the subsidy ceiling is much lower, being \$200/week, as opposed to \$350/week when unemployment

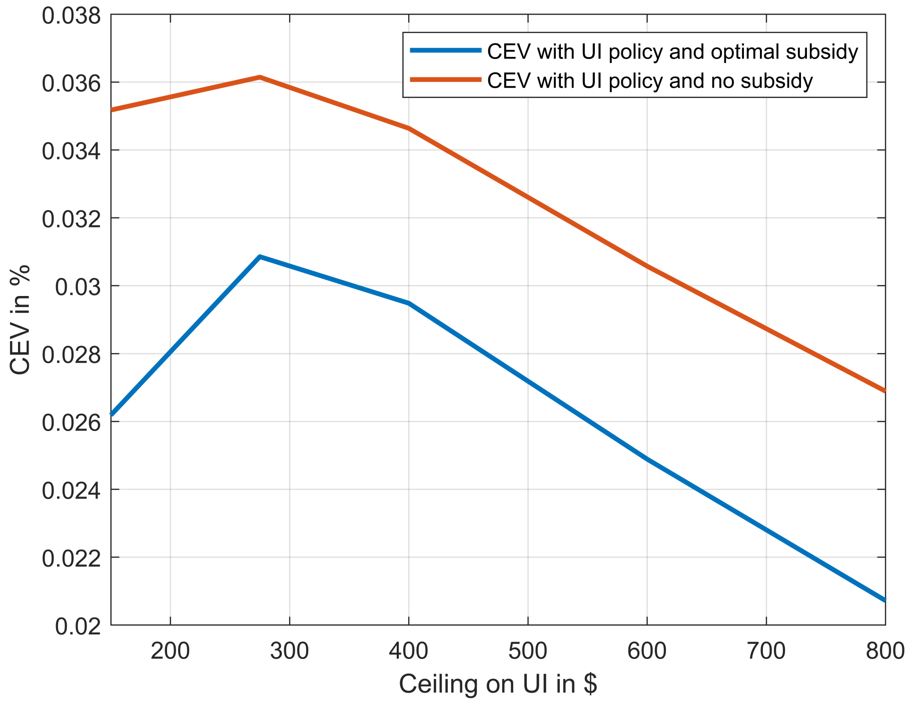

Figure 6: Consumption Equivalent Variation of the UI policy with various cap compared to the laissez-faire case

benefits are held constant. Second, the marginal subsidy rate is slightly higher when unemployment benefits are also adjusted. Overall, this means that the government offers weaker incentives to keep high earning workers on payroll when UI has been temporarily increased, and instead offers income replacement through benefits. This is because the extension of UI allows the policy maker to be more aggressive in avoiding subsidizing high profit jobs that would not have been displaced anyway because they are assured low income/wealth workers will collect higher unemployment benefits if they lose their jobs as a consequence.

4.5 The role of heterogeneity in policy making

The model features heterogeneity along several dimensions and the shutdown only hits the affected sector, thus any subsidy policy will naturally lead to redistribution. To determine which income groups gain the most from policy reforms, Figure 7 reports the results of a simple welfare decomposition. For both wage subsidy and UI policies the welfare gain is decreasing in income. For the lowest income group, the CEVs from wage subsidy and UI policy are about 0.055%. For other income groups, the CEV gains from the UI policy are higher than from the wage subsidy. Interestingly, the top income groups see a decrease in welfare under either policy. This is because (1) high income workers are more likely matched with high-productivity firms, and these firms would more likely retain their workers anyway, (2) high income workers often have high net worth and thus would be able to smooth consumption during the shutdown, and (3) both policies imply higher future tax rates with similar drops of λ to about $\tilde{\lambda} = .9012$, meaning long-run consumption falls by about 0.1%.

4.6 Further Robustness Checks

Here we report the results of two further robustness checks. The first check was to allow labor supply to respond to the increase in tax rates starting at time t^{rec}. We implement this in our model by shrinking each worker's value of θ by a fixed percentage. This fixed percentage is calculated by assuming a Frisch labor supply elasticity of 0.5, and using the reduction in net wages associated with the decrease of λ to $\tilde{\lambda}$ (note that the percentage reduction in net wage is independent of θ). To make this excercise feasible, we assume that labor supply falls permanently at $t = 0$, as opposed to t^{rec}, because otherwise there would be changes in the values of jobs and vacancies at t^{rec}, leading to further labor market transition. The main consequence of reducing labor supply is that the tax base is smaller in the new steady-state, implying a higher tax rate to pay for pandemic programs. However, the effect of reduced labor supply on taxes is very small, requiring $\tilde{\lambda} = 0.9011$ as opposed to $\tilde{\lambda} = 0.9012$. Similarly, the welfare gain from the optimal wage subsidy is smaller by only 4e-5%.

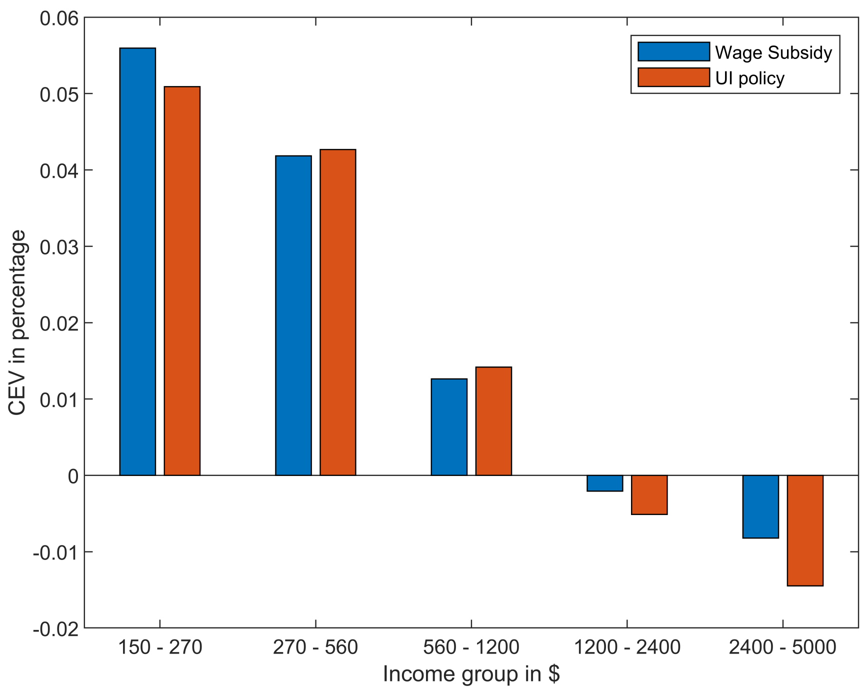

Figure 7: Decomposition of welfare gain from switching from baseline to optimal wage subsidy and UI policy.

Our final robustness check relates to how the labor market is calibrated. In our calibration the value of unemployment is quite low compared to employment because the replacement rate is only 40%. However, in Hagedorn and Manovskii (2008) the value of unemployment is much higher because of allowing for e.g. home production. We check the sensitivity to this by allowing home production equivalent to 50% of salary. In this case it is never optimal to subsidize wages during a pandemic, which makes sense because laid off workers can produce at home. We provide more details on this robustness check in Appendix C.

5 Conclusion

We have contributed a model that captures many relevant margins affecting the efficacy of employment related policies in a pandemic. The model includes labor market frictions that determine which workers will be laid off in a pandemic-related shut down, and also how quickly laid-off workers will find new jobs after the shut down ends. Wage rigidity in the model implies potentially large increases in the unemployment rate when the pandemic hits. Workers in the least profitable matches are the most likely to be laid off, and assortative matching between worker and job productivity implies that low income workers will be the hardest hit. The model also features an endogenous distribution of household consumption and wealth. Wealthy households can smooth consumption over the pandemic better than poorer households, which means that low income workers tend to also suffer the most from a layoff.

We considered wage subsidy and enhanced unemployment benefit policy responses to the economic disruption. Wage subsidies encourage firms to keep workers on payroll, which leads to a much faster labor market recovery after the pandemic ends. However, a wage subsidy program can be fiscally costlier than expanding unemployment benefits, implying a trade-off. Either policy option is paid for through higher future income tax rates.

We offer several concrete policy conclusions. Either a wage subsidy or enhanced unemployment benefits will reduce the welfare cost of the pandemic. Which of the two policy options is better depends on characteristics of the pre-pandemic economy and the nature of the pandemic. In an economy where the (pre-pandemic) average profit margin of firms is small (less than 12.25%), a wage subsidy program will be more beneficial than enhanced unemployment insurance, and vice-versa. This is because the low profit margins will imply large numbers of layoffs, absent a wage subsidy. Similarly, if the shut down period will be long (more than 11 weeks) then a wage subsidy is better. A long shut down cuts into the profit associated with keeping a worker on payroll, thus leading to large numbers of layoffs like when profit margins are small. Lastly, the most efficacious policy includes both a small wage subsidy and enhanced unemployment benefits. Under the baseline parameters of our model, calibrated to match U.S. data, a wage subsidy covering 90% of the first \$200/week of earnings combined with increased unemployment benefits replacing the first \$275/week of earnings is optimal. This is in contrast to the U.S. policy response, which expanded UI benefits by \$600 per week, but did not include a wage subsidy.

In closing, it is worth discussing several issues related to our study that we abstract from in our model. First, we do not model interactions between economic policy and disease transmission risk. This is because we take as given that workers in the affected sector cannot work. Thus, subsidizing their salaries keeps them on payroll, but does not change the likelihood of their being infected. Second, we abstract from long-run cleansing effects of recessions. Although jobs that are kept during the pandemic tend to be high productivity jobs, in the long run the distribution of job productivity returns to steady-state. Lastly, because we model wages as fixed, we do not model job-specific human capital. Such a feature would might add an additional layer of benefit to wage subsidies, as a labor market recovery would require not only full employment to be reattained, but also additional worker training. This might lengthen the recovery period, making future tax rates somewhat higher.

APPENDIX NOT FOR PUBLICATION

A Aggregates

The aggregate unemployment rate is:

$$U_t = \phi U_t^A + (1-\phi)U_t^{UA} \tag{26}$$

where

$$\begin{aligned} U_t^{UA} &= \sum_i \int_b \int_\theta g_t^{UA}(i,b,\theta,u)dbd\theta \\ U_t^{A} &= \sum_i \int_b \int_\theta \left(g_t^{A}(i,b,\theta,u) + \int_z \tilde{g}_t^A(i,b,\theta,a,z)dz \right) dbd\theta \end{aligned} \tag{27}$$

Next, we define some aggregates that are used to state the government budget constraint above. Aggregate tax revenue in the initial steady-state is:

$$\mathcal{T} = \sum_i \int_b \int_\theta T(\theta)g(i,b,\theta,e)dbd\theta + \sum_i \int_b \int_\theta \frac{T(\theta)}{\theta}h(\theta)g(i,b,\theta,u)dbd\theta \tag{28}$$

where $T(\theta)/\theta$ is the average tax rate of an individual with salary θ. In the final steady-state this becomes:

$$\tilde{\mathcal{T}} = \sum_i \int_b \int_\theta \tilde{T}(\theta)g^f(i,b,\theta,e)dbd\theta + \sum_i \int_b \int_\theta \frac{\tilde{T}(\theta)}{\theta}h(\theta)g^f(i,b,\theta,u)dbd\theta \tag{29}$$

where $g^f(i,b,\theta,s)$ is the distribution of household state variables in the final steady-state. During the pandemic, tax revenue is

$$\begin{aligned} \mathcal{T}_t =& \phi \sum_i \int_b \int_\theta \int_z T(\theta)\tilde{g}_t^A(i,b,\theta,e,z)dbd\theta dz + (1-\phi)\sum_i \int_b \int_\theta T(\theta)g_t^{UA}(i,b,\theta,e)dbd\theta \\ &+ \phi \sum_i \int_b \int_\theta \frac{T(\theta)}{\theta}\tilde{h}(\theta)(\int_z \tilde{g}_t^A(i,b,\theta,a,z)dz + g_t^A(i,b,\theta,u))dbd\theta \\ &+ (1-\phi)\sum_i \int_b \int_\theta \frac{T(\theta)}{\theta}h(\theta)g_t^{UA}(i,b,\theta,u)dbd\theta, \ 0 \le t < t^{open}. \end{aligned} \tag{30}$$

This simplifies during the recovery period to:

$$\begin{aligned}\mathcal{T}_t =& \sum_i \int_b \int_\theta T(\theta) g_t(i,b,\theta,e) dbd\theta \\ &+ \sum_i \int_b \int_\theta \frac{T(\theta)}{\theta} h(\theta) g_t(i,b,\theta,u) dbd\theta,\ t^{open} \le t < t^{rec}.\end{aligned} \tag{31}$$

In steady-state the aggregate cost of unemployment benefits is:

$$H = \sum_i \int_b \int_\theta h(\theta) g(i,b,\theta,u) dbd\theta \tag{32}$$

which is the same in both initial and final steady-states, given that the distributions of θ and s are the same in the two steady-states. During the pandemic the aggregate unemployment benefit cost is

$$\begin{aligned}H_t =& \phi \sum_i \int_b \int_\theta \tilde{h}(\theta)(\int_z \tilde{g}_t^A(i,b,\theta,a,z)dz + g_t^A(i,b,\theta,u)) dbd\theta \\ &+ (1-\phi) \sum_i \int_b \int_\theta h(\theta) g_t^{UA}(i,b,\theta,u) dbd\theta,\ 0 \le t < t^{open}\end{aligned} \tag{33}$$

while during the recovery period it is:

$$H_t = \sum_i \int_b \int_\theta h(\theta) g_t(i,b,\theta,u) dbd\theta,\ t^{open} \le t < t^{rec}. \tag{34}$$

During the pandemic, government expenditure on the wage subsidy is:

$$Q_t = \phi \sum_i \int_b \int_\theta \int_z \psi(\theta) \theta \tilde{g}_t^A(i,b,\theta,e,z) dbd\theta dz. \tag{35}$$

B Definition: Equilibrium

B.1 Initial Steady-State Equilibrium

An equilibrium for the initial steady-state is a value function $V(i, b, \theta, s)$ and optimal policy functions $c(i, b, \theta, s)$ and $\dot{b}(i, b, \theta, s)$, a probability measure $g(i, b, \theta, s)$, for $i \in \{1, 2, 3\}$ and $s \in \{e, u\}$, and a market tightness Θ such that

- Given market tightness Θ, the value function $V(i, b, \theta, s)$ solves the HJB equations (3) and (4), and $c(i, b, \theta, s)$ and $\dot{b}(i, b, \theta, s)$ are the associated policy function
- The market tightness Θ satisfies the free-entry condition (8) with $W = 0$
- The probably measure follows the KFE equation (5) with $\dot{g}(i, b, \theta, s) = 0$ for $i \in \{1, 2, 3\}$ and $s \in \{e, u\}$.

B.2 Final Steady-State Equilibrium

An equilibrium for the initial steady-state is a value function $V^f(i, b, \theta, s)$ and associated optimal policy functions $c^f(i, b, \theta, s)$ and $\dot{b}^f(i, b, \theta, s)$, a probability measure $g^f(i, b, \theta, s)$, for $i \in \{1, 2, 3\}$ and $s \in \{e, u\}$, and a market tightness Θ^f such that

- Given market tightness Θ^f, the value function $V^f(i, b, \theta, s)$ and associated policy functions $c^f(i, b, \theta, s)$ and $\dot{b}^f(i, b, \theta, s)$ solve augmented versions of the HJB equations (3) and (4) where the tax function $T(y)$ has been replaced by $\tilde{T}(y)$ that solves the final steady-state government budget constraint (19).
- The market tightness Θ^f satisfies the free-entry condition (8) with $W = 0$
- The probably measure follows the KFE equation (5) with $\dot{g}^f(i, b, \theta, s) = 0$ for $i \in \{1, 2, 3\}$ and $s \in \{e, u\}$.

B.3 Pandemic, Recovery and Post-Recovery Equilibrium

Let $\boldsymbol{s} = (i, b, \theta, s)$ be the usual worker state space, and $\boldsymbol{s}^A = (i, b, \theta, s, z)$ be the worker state space if they are in the affected sector during the pandemic phase. An equilibrium of the model is sequences of sector specific worker value functions $\{V_t^A(\boldsymbol{s}^A)\}_{0 \le t < t^{rec}}$ and $\{V_t^{UA}(\boldsymbol{s})\}_{0 \le t < t^{rec}}$, along with optimal policy functions $\{c_t^A(\boldsymbol{s}^A), \dot{b}_t^A(\boldsymbol{s}^A)\}_{0 \le t < t^{rec}}$ and $\{c_t^{UA}(\boldsymbol{s}), \dot{b}_t^{UA}(\boldsymbol{s})\}_{0 \le t < t^{rec}}$; sector specific job value functions $\{J_t^A(z, \theta)\}_{0 \le t < t^{open}}$ and $\{J_t^{UA}(z, \theta)\}_{0 \le t < t^{open}}$; sequences of general probability measures $\{g_t(\boldsymbol{s})\}_{t \ge t^{rec}}$ and sector specific probability measures $\{g_t^A(\boldsymbol{s}^A)\}_{0 \le t < t^{rec}}$

and $\{g_t^{UA}(\boldsymbol{s})\}_{0\leq t<t^{rec}}$; sequences of general market tightness $\{\Theta_t\}_{t\geq t^{rec}}$ and sector specific market tightness $\left\{\Theta_t^A\right\}_{0\leq t<t^{rec}}$ and $\left\{\Theta_t^{UA}\right\}_{0\leq t<t^{rec}}$; government policy functions $\left\{\tilde{T}, \tilde{h}(\theta), \psi(\theta)\right\}$; and government debt $\{B_t\}_{0\leq t\leq t^{rec}}$, such that, given initial and final steady-state equilibria:

- Given sequences of market tightness, government policy functions and V^f, the affected sector value functions $\{V_t^A(\boldsymbol{s}^A)\}_{0\leq t<t^{open}}$ solve equations (14), (15) and (16), and $\{V_t^A(\boldsymbol{s})\}_{t^{open}\leq t<t^{rec}}$ solve equations (20) and (16). $\{c_t^A(\boldsymbol{s}^A), \dot{b}_t^A(\boldsymbol{s}^A)\}_{0\leq t<t^{open}}$ and $\{c_t^A(\boldsymbol{s}), \dot{b}_t^A(\boldsymbol{s})\}_{t^{open}\leq t<t^{rec}}$ are the associated policy functions.

- Given sequences of market tightness, government policy functions and V^f, the unaffected sector value functions $\{V_t^{UA}(\boldsymbol{s})\}_{0\leq t<t^{rec}}$ solve variations of the HJB equations (3) and (4) as described in subsections 2.2.2 and 2.3.2, for example equation (17). $\{c_t^{UA}(\boldsymbol{s}), \dot{b}_t^{UA}(\boldsymbol{s})\}_{0\leq t<t^{rec}}$ are the associated policy functions.

- Given sequences of market tightness and government policy functions, the affected sector job value function $\{J_t^A(z,\theta)\}_{0\leq t<t^{open}}$ solves $J_t^A(z,\theta) = \max\left\{K_t^A(z,\theta), L_t^A(z,\theta)\right\}$ where $K_t^A(z,\theta)$ and $L_t^A(z,\theta)$ solve equations (11) and (12) given terminal condition $K_{t^{open}}^A(z,\theta) = L_{t^{open}}^A(z,\theta) = J^*(z,\theta)$.

- Given sequences of market tightness and government policy functions, the unaffected sector job value function $\{J_t^{UA}(z,\theta)\}_{0\leq t<t^{open}}$ solves equation (6) with J_t replaced by J_t^{UA}.

- The affected sector market tightness $\left\{\Theta_t^A\right\}_{0\leq t<t^{open}}$ satisfies the free-entry condition $W_t^A = 0$ for all such $t \in [0, t^{open})$, where W_t^A is defined in the RHS of (13).

- The affected sector market tightness $\left\{\Theta_t^A\right\}_{t^{open}\leq t<t^{rec}}$ satisfies the free-entry condition $W_t^A = 0$ for all such t, where W_t^A is defined in the RHS of (8) with J, g, W and U replaced by J_t^A, g_t^A, W_t^A and U_t^A, respectively.

- The unaffected sector market tightness $\left\{\Theta_t^{UA}\right\}_{0\leq t<t^{open}}$ satisfies the free-entry condition $W_t^{UA} = 0$ for all such t, where W_t^{UA} is defined in the RHS of (8) with J, g, W and U replaced by J_t^{UA}, g^{UA}, W_t^{UA} and U_t^{UA}, respectively.

- The general market tightness $\Theta_t = \Theta^f$ for all $t \geq t^{rec}$, where Θ^f satisfies the free-entry condition $W = 0$ for all such t, where W is defined in the RHS of (8).

- Government debt $\{B_t\}_{0\leq t\leq t^{rec}}$ satisfies equation (18) with $Q_t = 0$ for $t \geq t^{open}$, given $\psi(\theta)$, $\tilde{h}(\theta)$ and $B_0 = 0$.

- The reformed government tax function $\tilde{T}(y)$ satisfies (19), given the government debt $B_{t^{rec}}$

- The law-of-motion for the probability measure of unaffected workers for $t \in [0, t^{rec})$ is:

$$\begin{aligned}
\dot{g}_t^{UA}(i,b,\theta,e) =& -\partial_b[\dot{b}_t^{UA}(i,b,\theta,e)g_t^{UA}(i,b,\theta,e)] + f_t^{UA}g_t^{UA}(i,b,\theta,u) - \sigma g_t^{UA}(i,b,\theta,e) \\
&+ \sum_{j\neq i}\pi_\rho\left(g^{UA}(j,b,\theta,e) - g^{UA}(i,b,\theta,e)\right) \\
\dot{g}_t^{UA}(i,b,\theta,u) =& -\partial_b[\dot{b}_t^{UA}(i,b,\theta,u)g_t^{UA}(i,b,\theta,u)] - f_t^{UA}g_t^{UA}(i,b,\theta,u) + \sigma g_t^{UA}(i,b,\theta,e) \\
&+ \sum_{j\neq i}\pi_\rho\left(g^{UA}(j,b,\theta,u) - g^{UA}(i,b,\theta,u)\right)
\end{aligned} \tag{36}$$

- The probability measures of affected sector attached-unemployed and employed workers for $t \in [0, t^{open})$ satisfy the following conditions, where $I_t(z,\theta) = \mathcal{I}\left\{K_t(z,\theta) \geq L_t(z,\theta)\right\}$ and $dI_t(z,\theta) = \mathcal{I}\left\{K_t(z,\theta) \geq L_t(z,\theta)\right\}$-$\mathcal{I}\left\{K_{t-dt}(z,\theta) \geq L_{t-dt}(z,\theta)\right\}$:[15]

$$\begin{aligned}
\dot{\tilde{g}}_t^A(i,b,\theta,a,z) =&\ F(i,b,\theta,a,z) \quad \text{if } \ I_t(z,\theta) = 0 \\
\tilde{g}_t^A(i,b,\theta,a,z) =&\ 0 \quad \text{if } \ I_t(z,\theta) = 1
\end{aligned} \tag{37}$$

for attached-unemployed workers, where

$$\begin{aligned}
&F(i,b,\theta,a,z) = -\partial_b[\dot{b}_t^A(i,b,\theta,a,z)\tilde{g}_t^A(i,b,\theta,a,z)] \\
&- (\varphi + f_t^A P_z(z|\theta)\mathcal{I}\{K_t(z,\theta) \geq 0\})\tilde{g}_t^A(i,b,\theta,a,z) \\
&+ \sum_{j\neq i}\pi_\rho\left(\tilde{g}^A(j,b,\theta,a,z) - \tilde{g}^A(i,b,\theta,a,z)\right).
\end{aligned} \tag{38}$$

And for employed workers

$$\begin{aligned}
\dot{\tilde{g}}_t^A(i,b,\theta,e) =&\ \tilde{F}(i,b,\theta,e) & \forall\, t \in [0, t^{open}) \\
\tilde{g}_t^A(i,b,\theta,e) =&\ \tilde{g}_{t-dt}^A(i,b,\theta,a,z) + \tilde{g}_{t-dt}^A(i,b,\theta,e) + \dot{\tilde{g}}_{t-dt}^A(i,b,\theta,e)\cdot dt & \text{if } \ dI_t(z,\theta) = 1, \forall z
\end{aligned} \tag{39}$$

where

$$\begin{aligned}
&F(i,b,\theta,e) = -\partial_b[\dot{b}_t^A(i,b,\theta,e)\tilde{g}_t^A(i,b,\theta,e)] \\
&+ f_t^A \int_z P_z(z|\theta)\mathcal{I}\{K_t(z,\theta) \geq 0\}dz\left(\int_{\tilde{z}}\tilde{g}_t^A(i,b,\theta,a,\tilde{z})d\tilde{z} + g_t^A(i,b,\theta,u)\right) \\
&- \sigma\tilde{g}_t^A(i,b,\theta,e) \\
&+ \sum_{j\neq i}\pi_\rho\left(\tilde{g}^A(j,b,\theta,e) - \tilde{g}^A(i,b,\theta,e)\right).
\end{aligned} \tag{40}$$

[15]These conditions must account for the boundary conditions in the HJB equations, which generate jumps in the probability measures. Most importantly, at the moment $\mathcal{I}\left\{K_t(z,\theta) \geq L_t(z,\theta)\right\}$ switches "on" the measure of such attached unemployed workers is shifted to the measure of employed workers.

- The law-of-motion for the probability measure of affected sector unattached-unemployed workers for $t \in [0, t^{open})$ satisfies:

$$
\begin{aligned}
&\dot{g}_t^A(i,b,\theta,u) = -\partial_b[\dot{b}_t^A(i,b,\theta,u) g_t^A(i,b,\theta,u)] \\
&- \left(f_t^A \int_z P_z(z|\theta) \mathcal{I}\{K_t(z,\theta) \geq 0\} dz \right) g_t^A(i,b,\theta,u) \\
&+ \varphi \int_{\tilde{z}} \tilde{g}_t^A(i,b,\theta,a,\tilde{z}) d\tilde{z} \\
&+ \sigma \int_{\tilde{z}} \tilde{g}_t^A(i,b,\theta,e,\tilde{z}) d\tilde{z} \\
&+ \sum_{j \neq i} \pi_\rho \left(g^A(j,b,\theta,u) - g^A(i,b,\theta,u) \right)
\end{aligned} \tag{41}
$$

- The law-of-motion for workers in the affected sector for $t \in [t^{open}, t^{rec})$ satisfies:

$$
\begin{aligned}
\dot{g}_t^A(i,b,\theta,e) = &- \partial_b[\dot{b}_t^A(i,b,\theta,e) g_t^A(i,b,\theta,e)] + f_t^A g_t^A(i,b,\theta,u) - \sigma g_t^A(i,b,\theta,e) \\
&+ \sum_{j \neq i} \pi_\rho \left(g^A(j,b,\theta,e) - g^A(i,b,\theta,e) \right) \\
\dot{g}_t^A(i,b,\theta,u) = &- \partial_b[\dot{b}_t^A(i,b,\theta,u) g_t^A(i,b,\theta,u)] - f_t^A g_t^A(i,b,\theta,u) + \sigma g_t^A(i,b,\theta,e) \\
&+ \sum_{j \neq i} \pi_\rho \left(g^A(j,b,\theta,u) - g^A(i,b,\theta,u) \right)
\end{aligned} \tag{42}
$$

- $\lim_{t \to \infty} g_t = g^f$, and the law-of-motion for all workers for $t \in [t^{rec}, \infty)$ satisfies:

$$
\begin{aligned}
\dot{g}_t(i,b,\theta,e) = &- \partial_b[\dot{b}^f(i,b,\theta,e) g_t(i,b,\theta,e)] + f g_t(i,b,\theta,u) - \sigma g_t(i,b,\theta,e) \\
&+ \sum_{j \neq i} \pi_\rho \left(g(j,b,\theta,e) - g(i,b,\theta,e) \right) \\
\dot{g}_t(i,b,\theta,u) = &- \partial_b[\dot{b}^f(i,b,\theta,u) g_t(i,b,\theta,u)] - f g_t(i,b,\theta,u) + \sigma g_t(i,b,\theta,e) \\
&+ \sum_{j \neq i} \pi_\rho \left(g(j,b,\theta,u) - g(i,b,\theta,u) \right)
\end{aligned} \tag{43}
$$

C Robustness check: the value of outside option

Hagedorn and Manovskii (2008) argue that the value of outside option of unemployed workers is much higher than the standard calibration, amounting to 0.955. The outside option includes not only the unemployment benefit but also home production or leisure. We perform a robustness check along this line by assuming that the income of unemployed workers consists of two parts:

Table 3: CEV from the wage subsidy when the value of outside option is worth 95.55% percent of the employed income

Ceiling / Rate	20%	60%	100%
\$50	-0.0005%	-0.0011%	-0.0020%
\$250	-0.0020%	-0.0076%	-0.0161%
\$500	-0.0043%	-0.0184%	-0.0419%

a replacement rate of 40% paid by the government, and home production valued at 55% of the employment income. Both sources are taxed according to the HSV tax function, however the revenue from the home production is not used to fund the benefit programs. We make this assumption to ensure that the government debt in the robustness check is similar to the baseline mode, and so is the increase in taxes. We then solve the model for several combination of wage subsidy, and compute the associated CEV. We present the result in table (3) The CEV is negative for all considered combinations of wage subsidy parameters. This result makes sense because securing employment income for workers in the affected sector is the primary benefit of the wage subsidy, however with such a high value of outside options workers would have preferred being laid-off and enjoy a higher future consumption that they would have otherwise if they had paid for the wage subsidy.

D Distribution of individual skill θ

We use data from the March 2019 Current Population Survey Annual Social and Economic Supplement (CPS). We choose this period as it is the nearest date prior to the Covid-19 pandemic, i.e. the 2020 supplement may include some effects from the pandemic. To construct weekly wages, we divide annual earnings by the number of weeks worked. We then compute the mean and variance of log-weekly earnings, including only those between the ages of 21 and 65.

E Details of algorithm

E.1 Solving the steady-state HJB equation

The model is solved using a finite-difference method proposed by Achdou, Han, Lasry *et al.* (2017). Besides the computational advantage of this formulation, the continuous time model adds an extra element of realism that captures the job-search process in the real world: in discrete-time models households have to wait for one period (for example, one period is 6 weeks in KMS) to find a job. In the continuous-time model, conditional on the same job-finding probability as in the discrete-time counterpart, households will have a good chance to find a job in a short lapse of time[16]. This difference also between discrete and continuous-time formulation also affects the precautionary motive of households, which is an important aspect that we would like to capture when studying the household welfare under different policy designs.

We discretize the state-space into a fine grid, and then approximate the value functions at a discrete points in the grid. We first discuss how to solve the initial steady-state HJB equation 3, which is the same for both sectors. Let $b_i, i = 1, ..., I$ denote the discrete point in the asset dimension. We use a non-uniform grid and concentrate more points on the bottom part of the asset grid where most of the concavity arises. Let ρ_j denotes the time preference with $j \in \{1, 2, 3\}$. Let $\Lambda_{s,s'}$ denote the Poisson rate of switching from state s to s'. For example, if $s = e$ then $s' = u$, and $\Lambda_{s,s'} = \sigma$ i.e the job separation rate. Let $\theta_k, k = 1, ..., K$ denote the discrete point in the skill dimension. Denote the argument of a function by the subscript. For example, $V_{i,s,j}$ is $V(b_i, s, \rho_j)$ i.e the value function at each discrete point in the state-space. Let y_s denote the after-tax labor / unemployment benefit income, which only depends on the labor status. We suppress the dependence of value function on θ, with the understanding that θ will enter the the value function through labor income and unemployment benefit. The discretized steady-state HJB equation is:

$$\rho_j V_{i,s,j} = u(c_{i,s,j}) + \underbrace{\frac{V_{i+1,s,j} - V_{i,s,j}}{\Delta b}}_{\partial_b V^F_{i,s,j}}[\dot{b}_{i,s,j}]^+ + \underbrace{\frac{V_{i,s,j} - V_{i-1,s,j}}{\Delta b}}_{\partial_b V^B_{i,s,j}}[\dot{b}_{i,s,j}]^- + \Lambda_{s,s'}[V_{i,s',j} - V_{i,s,j}] + \sum_{h \neq j} \pi_\rho [V_{i,s,h} - V_{i,s,j}] \tag{44}$$

where for any x we have $[x]^+ = \max(0, x)$ and $[x]^- = \min(0, x)$. The upwind scheme means if the asset drift is positive then we will use a forward difference, and if the asset drift is negative we will use a backward difference. Otherwise, if the drift is zero i.e we are in the steady-state, the central difference will be used.

The FOC for 3 is $u'(c) = \partial_b V \iff c = (\partial_b V)^{-\frac{1}{\gamma}}$ for the CRRA utility function, and it is held everywhere. As suggested by Achdou, Han, Lasry *et al.* (2017), the borrowing constraint is replaced by a *state constraint boundary condition*:

$$V_{s,j}(\underline{b}) \geq u'(y_s + r\underline{b}), \forall s, j \tag{45}$$

[16]This insight is taken from Bence Badoczy's note on solving KMS in continuous time

Now, define the consumption policy associated with the positive, negative, and no drift as follows:

$$
\begin{aligned}
c^F_{i,s,j} &= (\partial_b V^F_{i,s,j})^{-\frac{1}{\gamma}} \\
c^B_{i,s,j} &= (\partial_b V^B_{i,s,j})^{-\frac{1}{\gamma}} \\
c^o_{i,s,j} &= y_s + rb_i
\end{aligned}
\tag{46}
$$

where the last equation is derived from the budget constraint with $\dot{b} = 0$. The saving decisions in turns are:

$$
\begin{aligned}
\dot{b}^F_{i,s,j} &= y_s - rb_i - c^F_{i,s,j} \\
\dot{b}^B_{i,s,j} &= y_s - rb_i - c^B_{i,s,j}
\end{aligned}
\tag{47}
$$

The final consumption policy using the upwind scheme is as follows:

$$
c_{i,s,j} = \dot{c}^F_{i,s,j}\mathcal{I}\{b^F_{i,s,j} > 0\} + \dot{c}^B_{i,s,j}\mathcal{I}\{\dot{b}^B_{i,s,j} < 0\} + c^0_{i,s,j}\mathcal{I}\{\dot{b}^F_{i,s,j} < 0 < \dot{b}^B_{i,s,j}\} \tag{48}
$$

where the last term is derived from the fact that since V is concave in b we have:

$$
\partial_b V^F_{i,s,j} < \partial_b V^B_{i,s,j} \implies \dot{b}^F_{i,s,j} < \dot{b}^B_{i,s,j} \tag{49}
$$

which gives us three distinct cases listed above.

Let n index the iterations of the value function. HJB in the matrix form, using an implicit method, is:

$$
\begin{aligned}
&\frac{\boldsymbol{V}^{n+1} - \boldsymbol{V}^n}{\Delta} + \boldsymbol{\rho}\boldsymbol{V}^{n+1} = u(\boldsymbol{c}^n) + (\boldsymbol{B}^n + \boldsymbol{\Lambda}^n + \boldsymbol{\Pi})\boldsymbol{V}^n \\
\iff &\boldsymbol{V}^{n+1} = \left[\frac{1}{\Delta}\boldsymbol{I} + \boldsymbol{\rho} - \boldsymbol{B}^n - \boldsymbol{\Lambda}^n - \boldsymbol{\Pi}\right]^{-1}\left(u(\boldsymbol{c}^n) + \frac{1}{\Delta}\boldsymbol{V}^n\right)
\end{aligned}
\tag{50}
$$

Here, $\boldsymbol{I}$ is an identity matrix of size $I \times 2 \times 3 \times K$, $\boldsymbol{\rho}$ is a vector stacking $[\rho_1, \rho_2, \rho_3]$, $\boldsymbol{c}$ is a vector stacking the optimal consumption policy, $\boldsymbol{V}$ is a vector stacking the value function, and $\boldsymbol{B}^n$ captures the transition along the asset dimension, and it is defined as:

$$
\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_{e,j} & \boldsymbol{0} \\ 0 & \boldsymbol{B}_{u,j} \end{bmatrix} \tag{51}
$$

and for $s \in \{e, u\}$ and $j \in \{1, 2, 3\}$:

$$\boldsymbol{B}_{s,j} = \begin{bmatrix} y_{1,s,j} & \upsilon_{1,s,j} & 0 & 0 & \cdots & 0 \\ x_{2,s,j} & y_{2,s,j} & \upsilon_{2,s,j} & 0 & \cdots & 0 \\ 0 & x_{3,s,j} & y_{3,s,j} & \upsilon_{3,s,j} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & x_{I,s,j} & y_{I,s,j} \end{bmatrix} \tag{52}$$

where:

$$\begin{aligned} x_{i,s,j} &= -\frac{[\dot{b}_{i,s,j}]^-}{\Delta b} \\ \upsilon_{i,s,j} &= -\frac{[\dot{b}_{i,s,j}]^+}{\Delta b} \\ y_{i,s,j} &= -x_{i,s,j} - \upsilon_{i,s,j} \end{aligned} \tag{53}$$

The intuition is that in continuous time, workers can only change its asset position by the size amounting to one step in the asset grid. An implication of this is that the matrix $\boldsymbol{B}^n$ is very sparse.

$\boldsymbol{\Lambda}^n$ captures the transition along employment status dimension, and it is defined as follows:

$$\boldsymbol{\Lambda}^n = \begin{bmatrix} -\boldsymbol{\sigma} & \boldsymbol{\sigma} \\ \boldsymbol{f} & -\boldsymbol{f} \end{bmatrix} \tag{54}$$

$\boldsymbol{\Pi}$ captures the transition along the time preference dimension:

$$\boldsymbol{\Pi} = \begin{bmatrix} -2\boldsymbol{\pi} & \boldsymbol{\pi} & \boldsymbol{\pi} \\ \boldsymbol{\pi} & -2\boldsymbol{\pi} & \boldsymbol{\pi} \\ \boldsymbol{\pi} & \boldsymbol{\pi} & -2\boldsymbol{\pi} \end{bmatrix} \tag{55}$$

where the transition rate from ρ_j to ρ_h is the same for any $h \neq j$.

Finally, since θ is fixed skilled, there is no transition along this dimension.

With any initial guess $V^n_{i,s,j}$, iterating the equation 50 will eventually result in a convergence i.e $\lim_{n\to\infty} \max ||V^{n+1}_{i,s,j} - V^n_{i,s,j}|| = 0$. The intuition is that starting from any value function, if workers experience through the asset accumulation and labor uncertainty for long enough then their decision making will eventually be stationary. Also, the sparsity of transition matrix implies that such iterating method could be solved very efficiently in a matrix-oriented language (e.g Matlab).

E.2 Solving the transitional-dynamics HJB equation

We first discuss how to solve the sequence of value functions for unaffected sector. The only variable that generate the dynamics in the choice of unaffected workers is the future labor tax. Let

$V^n_{i,s,j} = V(b_i, s, \rho_j, t^n)$, where t^n is the discrete point in time space. The transitional-dynamics HJB equation is:

$$\rho_j V^n_{i,s,j} = u(c^n_{i,s,j}) + \underbrace{\frac{V^n_{i+1,s,j} - V^n_{i,s,j}}{\Delta b}}_{\partial_b V^F_{i,s,j}} [\dot{b}^n_{i,s,j}]^+ + \underbrace{\frac{V^n_{i,s,j} - V^n_{i-1,s,j}}{\Delta b}}_{\partial_b V^B_{i,s,j}} [\dot{b}^n_{i,s,j}]^-$$
$$+ \Lambda_{s,s'}[V^n_{i,s',j} - V^n_{i,s,j}] + \sum_{h \neq j} \pi_\rho [V_{i,s,h} - V_{i,s,j}] + \underbrace{\frac{V^{n+1}_{i,s,j} - V^n_{i,s,j}}{\Delta t^n}}_{\partial_t V_{i,s,j}} \tag{56}$$

where the optimal policy functions are time-variant is now also a function of the taxation scheme. In particular:

$$\dot{b}^n_{i,s,j} = y_s(\tau^n) - r b^n_i - c^n_{i,s_j} \tag{57}$$

where the policy-dependence of optimal policy is captured in $y_s(\tau^n)$ which depends on time. We solve this HJB equation backward, starting from the terminal stationary value function V^N_{i,s_j} that solves the 44 with the reformed federal tax $\tilde{\tau}$ in place. The transitional-dynamics HJB equation in the matrix form, using an implicit method, is:

$$\boldsymbol{V}^n = \left[\frac{1}{\Delta}\boldsymbol{I} + \boldsymbol{\rho} - \boldsymbol{B}^n - \boldsymbol{\Lambda}^n - \boldsymbol{\Pi}\right]^{-1} \left(u(\boldsymbol{c}^n) + \frac{1}{\Delta}\boldsymbol{V}^{n+1}\right) \tag{58}$$

There are two differences between 50 and 58. First, the time-variant parameters show up in the latter but not the former. Second, the time index is reversed (notice the blue highlight). Intuitively, in the transitional dynamics we know the end value function, and we iterate from that and goes backward to the ”current time”. In the steady-state equilibrium, we do not know what the end point is, thus we iterate from any guess of the value function, and go forward until we reach the steady-state.

We now turn to solve the sequence of value functions for affected sector. The labor status is now $s \in \{z, u\}$. Let $z_j, j = 1, ..., J$ be the discrete point in the job-specific productivity space. The main difference between this problem and the unaffected workers problem is that workers can be in multiple employment states: (1) employed at job z, (2) attached-unemployed with job z, and (3) unemployed. The transition matrix along the labor status dimension is:

$$\boldsymbol{\Lambda}^n = \begin{bmatrix} \boldsymbol{EE} & \boldsymbol{EA} & \boldsymbol{EU} \\ \boldsymbol{AE} & \boldsymbol{AA} & \boldsymbol{AU} \\ \boldsymbol{UE} & \boldsymbol{UA} & \boldsymbol{UU} \end{bmatrix} \tag{59}$$

Where $\boldsymbol{EU}$ is a matrix describing the employment-unemployment transition, and so forth. These submatrices describes the law-of-motion as laid down in 40, 38 and 41

E.3 Characterizing the law-of-motion of the probability measure

As shown in Achdou, Han, Lasry *et al.* (2017), the law-of-motion for the probability measure g can be characeterized by the following Kolmogorov forward equation:

$$\dot{g}_t = [\boldsymbol{B}^n + \boldsymbol{\Lambda}^n + \boldsymbol{\Pi}]^\top g_t \tag{60}$$

where the transition matrix is time-independent if we solve for the steady-state, and it is time-dependent if we solve for the transitional dynamics. In the steady state, $\dot{\boldsymbol{g}}_t = 0$ and thus

$$0 = [\boldsymbol{B} + \boldsymbol{\Lambda} + \boldsymbol{\Pi}]^\top g \tag{61}$$

Once we construct $\boldsymbol{B}$ and $\boldsymbol{\Lambda}$ for the HJB, we get the stationary distribution almost for free[17]. We then paste the distribution of θ into g and rescale so that it integrates to one.

When we compute the aggregate variables along the transitional dynamics, we use the stationary measure associated with the initial steady state to initialize the chain 60.

[17] While 61 can be solved by the slash operator in Matlab, we find that iterating 60 is more stable

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