Central Bank Digital Currency and Financial Fragility*

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Abstract

How does the introduction of a remunerated Central Bank Digital Currency (CBDC) affect financial stability? To study this issue, we introduce CBDC in a model in which a bank attracts deposits and is subject to runs, whose probability is endogenously pinned down via global-games methods. We first validate a commonly held view that higher CBDC remuneration increases the withdrawal incentives of investors and thus bank fragility. Second, we identify a contrarian force: the bank raises deposit rates in response to higher CBDC remuneration to retain deposits that, in turn, reduces fragility. These opposing effects can lead to a U-shaped relation between CBDC remuneration and bank fragility. Finally, we examine CBDC holding limits and their impact on fragility, deposit rates, and optimal CBDC remuneration.

Keywords: Central Bank Digital Currency, Bank Fragility, Global Games.

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*The views expressed in this paper are the authors' and do not necessarily reflect those of the European Central Bank or the Eurosystem.

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1 Introduction

The costs and benefits of issuing a central bank digital currency (CBDC) are currently being researched by the majority of the world's central banks (Boar and Wehrli, 2021). This is in response to the concern that digitalization of commerce will reduce the demand for physical currency—the only form of central bank money presently available to the public—in the same way as the internet and email reduced demand for the postage stamp (Panetta, 2021). In an increasingly digital economy, CBDC could ensure that central bank money continues to play an important role as a means of payment and store of value. The wider economic implications of the introduction of CBDC are at the center of a considerable debate among academics and policymakers.

One issue of particular interest is the effect of CBDC on financial stability. Its status as safe asset with potentially positive remuneration—a key difference to physical cash—could render it an attractive store of value and may increase the risk of bank runs during crisis episodes (BIS, 2021). To mitigate such concerns, policy makers have proposed restrictions such as holding limits and tiered remuneration schedules (Bindseil et al., 2021). However, these discussions are at an early stage, since most CBDC projects are currently in the investigation phase.[1]

This paper aims to inform this debate by developing a two-period bank-run model with remunerated CBDC. At an initial date a bank with access to profitable but risky investment opportunities attracts uninsured deposits from investors. At an interim date (i.e. before the maturity of investment), investors receive a noisy private signal about the profitability of investment (and thus about the solvency of the bank) and decide whether to withdraw their funds or roll them over. The introduction of a remunerated CBDC has two effects in this economy: first, it improves the outside option of investors when the bank raises funding. Second, it increases the return to investors withdrawing from the bank and depositing with the central bank for consumption at the final date.

[1] To date, only the Bahamas and Nigeria have launched a CBDC.

We use global-games methods to pin down a unique equilibrium at the withdrawal stage (Carlsson and van Damme, 1993; Morris and Shin, 2003; Vives, 2005). We follow the particular setup of Goldstein and Pauzner (2005) and Carletti et al. (2022) to derive a unique threshold of investment profitability (economic fundamentals) below which the bank fails (Proposition 1). This approach allows us to study how the deposit contract and CBDC remuneration affects bank fragility.

Consistent with the ongoing policy debate, we show that—for a given deposit contract—an increase in CBDC remuneration increases the probability of a bank run (Proposition 2). Ceteris paribus, higher CBDC remuneration increases the incentives to withdraw from the bank before investment matures.

In equilibrium, however, a profit-maximizing bank adjusts the contract terms in order to retain deposits. A higher deposit rate, in turn, reduces depositor incentives to withdraw (Proposition 3). Accordingly, the overall effect of CBDC on financial stability depends on which of these two forces dominates. Contrary to the received wisdom, we show that the introduction of a remunerated CBDC can actually improve financial stability (Proposition 4). For some parameters, we obtain a U-shaped relationship between bank fragility and CBDC remuneration.

A numerical example suggests that the level of CBDC remuneration that maximizes financial stability is about 4.9% per period. While this number may seem high, it implies roughly a 10% rate over two periods. The two periods in the model are not two years, rather they corresponds to the average maturity of bank investment (i.e. several years), resulting in a plausible magnitude for yearly CBDC remuneration.

In ongoing work, we study the implications of holding limit design for deposit rates, bank fragility and the fragility-minimising level of CBDC remuneration.

Extensions. To probe the robustness of our results, we consider three extensions. First, we study a version of our model without costly liquidation of investment such that no panic runs on the bank exist and all bank failures are driven by fundamental insolvency only. We find that higher CBDC remuneration always decreases the relevant measure of financial stability (Proposition 5). This result suggests that panic runs are an essential ingredient to our main result.

Second, we wish to explore the role of market power in the deposit market. We have so far assumed a monopolist bank (Andolfatto, 2021), which can respond quite strongly to the introduction of CBDC, a competition for its deposits, by increasing deposit rates. However, when there are multiple banks competing for deposits, the ability of a given bank to attract or retain funding by raising rates may be more limited, suggesting a more prominent role of the direct effect of CBDC remuneration on bank fragility (resulting in lower financial stability). We seek to explore this issue in a spatial model of deposit competition (Salop, 1979) in future work.

Third, we have considered a fragile liability side of banks (uninsured deposits) as a source of financial instability so far. A large literature in banking is concerned with the risk-taking of banks on their asset side (e.g., risk choices and asset substitution). Our setup can be naturally extended along this dimension. Since the banker has to raise deposit rates to retain deposit funding in response to higher CBDC remuneration, it is less profitable. Such lower skin in the game would exacerbate a moral hazard problem in risk choices, contributing to financial instability. We plan to formally investigate this additional channel in future work.

2 Model

An economy extends over three dates $t = 0, 1, 2$ and is populated by a bank and a unit continuum of investors $i \in [0, 1]$. There is a single divisible good for consumption and investment. All agents are risk neutral and do not discount the future. Investors are endowed with one unit of funds at date 0 only.

At date 0, the bank has access to a profitable but risky investment technology. To finance investment, the bank raises funds from investors in exchange for demandable-deposit contracts.[2] Investment returns $L \in (0, 1)$ if liquidated at date 1 (the liquidation value) and $R\theta$ upon maturity at date 2, where $\theta \sim U[0, 1]$ represents the fundamentals of the economy and $R > 2$ is a constant that reflects the return from lending (or from financial intermediation more broadly).

The deposit contract specifies a repayment $r_1 \geq 1$ at date 1 and r_2 at date 2. Thus it gives investors the option to withdraw before the maturity of investment. This decision is based on a noisy private signal about the fundamental at date 1:

$$s_i = \theta + \varepsilon_i, \tag{1}$$

with $\varepsilon_i \sim U[-\varepsilon, +\varepsilon]$. The signal gives investors information about the fundamental θ as well as the signals (and withdrawal actions) of other investors. As is standard in much of the global-games literature, we assume vanishing noise, $\varepsilon \to 0$, to simplify the analysis of date 0 choices and the implications of CBDC.

The bank satisfies interim withdrawals by liquidating investment. Let $n \in [0, 1]$ be the fraction of investors who withdraw at date 1. When the liquidation proceeds at date 1 are insufficient to meet withdrawals, $n > \overline{n} \equiv \frac{L}{r_1}$, the bank is

[2]Bank debt is assumed to be demandable. Demandability arises endogenously with liquidity needs (Diamond and Dybvig, 1983) or as a commitment device to overcome an agency conflict (Calomiris and Kahn, 1991; Diamond and Rajan, 2001). Accordingly, uninsured deposits refer to any short-term or demandable debt instrument, which includes uninsured retail deposits and insured deposits when deposit insurance is not credible (e.g., when the funding of the deposit insurance fund is insufficient). Three quarters of U.S. commercial bank funding are deposits and in the largest commercial banks, half of which are uninsured (Egan et al., 2017).

bankrupt due to illiquidity. Otherwise, the bank continues until date 2. If at date 2 it cannot meet the remaining withdrawals, $n > \widehat{n} \equiv \frac{R\theta - r_2}{R\theta \frac{r_1}{L} - r_2}$, it is bankrupt due to insolvency, where $\widehat{n}$ solves the insolvency condition

$$R\theta \left(1 - \frac{\widehat{n} r_1}{L}\right) = (1 - \widehat{n})\, r_2. \tag{2}$$

Bankruptcy is costly and we assume zero recovery for simplicity.[3]

Finally, we introduce CBDC. A deep-pocketed central bank offers investors deposits with a return $\omega \in [1, R)$ per period. Thus, the introduction of CBDC has two effects. First, it improves the outside option of investors deciding whether to deposit with the bank at date 0: from 1 to ω^2, which is the compounded return earned over two periods from date 0 to date 2. Second, it offers an interest ω between dates 1 and 2 to those depositors who decide to withdraw at date 1.[4]

3 Equilibrium

We start by characterizing the threshold $\theta^* = \theta^*(r_1, r_2; \omega)$ below which the bank fails and how it depends on the deposit contract and CBDC remuneration. Next, we solve for the optimal bank deposit contract (r_1^*, r_2^*) and its dependence on CBDC remuneration ω. Finally, we show how CBDC remuneration affects bank fragility, $\frac{d\theta^*}{d\omega}$, once the indirect effect via deposit rates $r_2^*(\omega)$ is taken into account.

As a preliminary step, we note that the bank chooses a deposit contract such that

$$\omega r_1^* < r_2^* < R \tag{3}$$

in any equilibrium. If $\omega r_1 \geq r_2$, then there would always be a run on the bank, resulting zero expected profits, a contradiction. Moreover, deposit-taking cannot be profitable if $r_2 \geq R$, so they bank would not choose to offer such a deposit rate.

[3]The magnitude of bankruptcy costs is large. For example, James (1991) estimated to be 30 cent on a dollar, making bank failures socially costly.

[4]We abstract from both raising funds (e.g. via taxation) and an investment choice at $t = 0$.

3.1 Bank fragility

We use global-games methods to solve for the unique equilibrium at the withdrawal stage. To characterize an individual investor's withdrawal decision, we start with the dominance bounds. First, as in Goldstein and Pauzner (2005), we assume that when $\theta \to 1$, the liquidation value is high, $L = R$, such that it is a dominant strategy not to withdraw. We denote this range as the upper dominance region $[\overline{\theta}, 1]$, where the (upper dominance) bound $\overline{\theta}$ can be arbitrarily close to 1. In fact, we assume $\overline{\theta} \to 1$ for the analysis of the choice of deposit contract at date 0. Second, withdrawing is a dominant strategy when $\theta < \underline{\theta}$, where the (lower dominance) bound $\underline{\theta}$ solves

$$R\underline{\theta} - r_2 = 0, \tag{4}$$

so that $\underline{\theta} = \frac{r_2}{R} \in (0, 1)$. Let us gather some intuition for this result. When no other investor withdraws, an investor is sure to receive $r_2 > \omega r_1$ when the bank is solvent at date 2, i.e. when $R\theta \geq r_2$. Otherwise, an investor receives 0 if she waits and r_1 if she withdraws, which is worth $\omega\, r_1$ at date 2 when the withdrawal proceeds are deposited with the central bank between date 1 and 2. Thus, withdrawing is a dominant action when $R\theta < r_2$, that is for $\theta < \underline{\theta}$.

In the intermediate range $(\underline{\theta}, \overline{\theta})$, an investor's decision to withdraw depends on what she expects the other investors do. Using global-games techniques, we state the bank failure threshold at date 1, which we refer to as bank fragility θ^*.

Proposition 1. ***Failure threshold.*** *There exists a unique fundamental threshold* $\theta^* \in \left(\underline{\theta}, \overline{\theta}\right)$. *Each investor withdraws their deposits from the bank if and only if* $\theta < \theta^*$, *where*

$$\theta^*(r_1, r_2) \equiv \underline{\theta}\frac{r_2 - \omega L}{r_2 - \omega r_1} > \underline{\theta}. \tag{5}$$

Proof. See Appendix A. □

Having established a unique equilibrium allows us to study how CBDC remuneration and the deposit contract affect bank fragility.

Proposition 2. *Fragility is a convex function of long-term deposit rates:* $\frac{\partial \theta^*}{\partial r_2} < 0$ *if and only if* $r_2 < r_2^{max} \equiv \omega r_1 \left(1 + \sqrt{1 - \frac{L}{r_1}}\right)$. *Higher CBDC remuneration directly increases fragility,* $\frac{\partial \theta^*}{\partial \omega} > 0$. *Liquidity provision increases fragility,* $\frac{\partial \theta^*}{\partial r_1} > 0$, *while better investment characteristics reduce fragility,* $\frac{\partial \theta^*}{\partial L} < 0$ *and* $\frac{\partial \theta^*}{\partial R} < 0$.

Proof. See Appendix A. □

Figure 1 shows the non-monotonic relationship between the long-term deposit rate and bank fragility: when the deposit rate is low, higher rates reduces fragility while the opposite holds for high deposit rates (U-shaped).

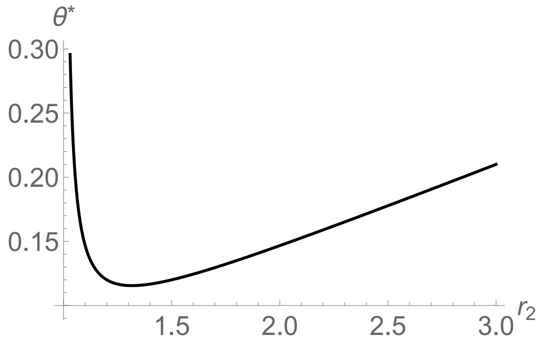


Figure 1: Bank failure threshold θ^* and the long-term deposit return r_2. The minimum is reached at $r_2 = r_2^{max}$. Parameters: $L = 0.9$, $R = 15$, $\omega = 1$; $r_1 = 1$.

We also obtain the standard result of asset illiquidity and liquidity provision resulting in bank fragility (Diamond and Dybvig, 1983). Bank liquidity provision, $r_1 > L$, results in strategic complementarity in investor withdrawal decisions, so both panic runs and fundamental runs exist. This results in bank fragility, $\theta^* > \underline{\theta}$.

3.2 Bank deposit rates

We next derive the optimal deposit rates. Recall that private noise and the upper dominance region vanish, $\epsilon \to 0$ and $\overline{\theta} \to 1$, so the banker's problem at date 0 is

$$\max_{r_1 \geq 1, r_2} \Pi \equiv \int_{\theta^*}^{1} (R\theta - r_2)\, d\theta = (1 - \theta^*) \left(\frac{R}{2}(1 + \theta^*) - r_2 \right)$$

subject to

$$V \equiv \int_{\theta^*}^{1} r_2 \, d\theta \geq \omega^2, \qquad \text{and } \theta^* = \theta^*(r_1, r_2).$$

Because of vanishing noise, $\epsilon \to 0$, no investor withdraws for $\theta > \theta^*$ and all investment matures at date 2, yielding $R\theta$. The banker pays the promised return r_2 to a unit mass of investors that roll over their funding for $\theta > \theta^*$ and keeps the difference, $R\theta - r_2$. Conversely, for $\theta < \theta^*$, all investors withdraw and the bank is bankrupt, resulting in a loss of output due to costly bankruptcy. The banker is protected by limited liability and receives zero in this case.

Taken together, we obtain the expected bank profits Π at date 0 and the expected value of the deposit claim V at date 0. The outside option of investors is not to fund the bank but to deposit in CBDC with the central bank, earning a return ω per period. Expected bank profits have a natural interpretation. The first term is the probability of no bank run and the second term is the expected bank profits conditional on no bank run. Note, finally, that the bank internalizes how its choices of deposit rates influence fragility, $\theta^* = \theta^*(r_1, r_2)$.

Our first result on the optimal short-term deposit rate r_1^* follows. Since a higher r_1 increases fragility (Proposition 2), it reduces the objective function because the bank is solvent less often, $\frac{\partial \Pi}{\partial r_1} = (R\theta^* - r_2)\frac{\partial \theta^*}{\partial r_1} < 0$, and tightens the constraint because investors are repaid less often, $\frac{\partial V}{\partial r_1} = -r_2 \frac{\partial \theta^*}{\partial r_1} < 0$. The former inequality arises because bank fragility is costly in terms of bank value: in other words, the equity value at the failure threshold is positive, $(R\theta^* - r_2) = r_2 \omega \frac{(r_1 - L)}{r_2 - \omega r_1} > 0$. The next result follows directly from these arguments.

Lemma 1. ***Optimal short-term deposit rate.*** *The banker sets* $r_1^* = 1$.

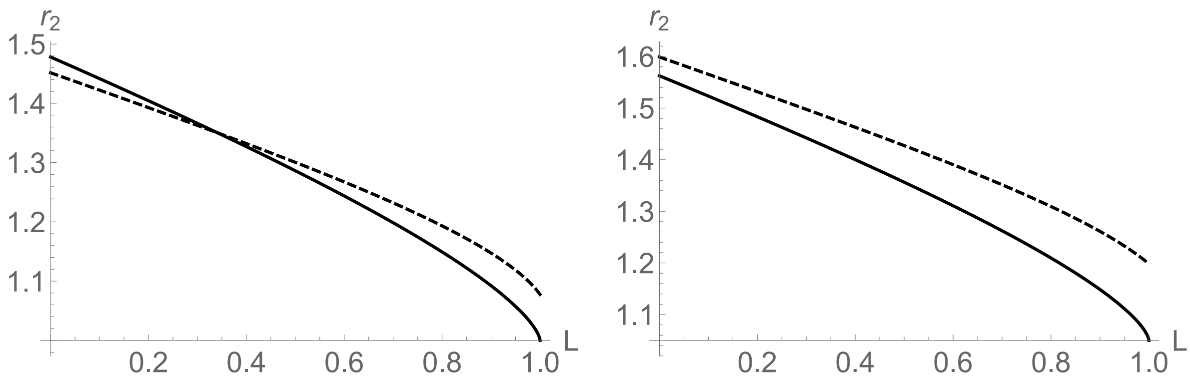
We are now ready to state our main result on bank deposit rates.

Proposition 3. ***Optimal long-term deposit rate.*** *For* $L \geq \underline{L}$, *the optimal deposit rate* $r_2^* < r_2^{max}$ *corresponds to the solution to the binding participation constraint of investors,* $V(r_2^*) \equiv \omega^2$. *The solution* r_2^* *increases with CBDC remuner-*

ation, while it decreases with the liquidation value and profitability of investment: $\frac{\partial r_2^*}{\partial \omega} > 0$*,* $\frac{\partial r_2^*}{\partial L} < 0$*, and* $\frac{\partial r_2^*}{\partial R} < 0$*.*

Proof. See Appendix B, which also defines $\underline{L}$. $\square$

In general, the optimal deposit rate r_2^* is pinned down by either the first-order condition of the bank (zero marginal profits) or by a binding participation constraint of investors. Figure 2 shows that there exists (high enough) liquidation values of investment for which the participation constraint determines the equilibrium deposit rate. In what follows, we assume a high enough value, $L \geq \overline{L}$, and a binding participation constraint in equilibrium.



(a) The graph is drawn for the following parameters: $R = 15$ and $\omega = 1$

(b) The graph is drawn for the following parameters: $R = 15$ and $\omega = 1.05$

Figure 2: Optimal deposit rate r_2^* and the liquidation value L. The shaded lines shows the deposit rate implied by a binding participation constraint, while the solid line shows the deposit rate implied by zero marginal profits of the bank. The equilibrium deposit rate r_2^* is the upper envelope of both curves. Panel (a) shows that the participation constraint binds for high enough values of L. Panel (b) shows that the participation constraint always binds.

A higher liquidation value of investment or higher investment profitability both reduce bank fragility ($\frac{\partial \theta^*}{\partial L} < 0$ and $\frac{\partial \theta^*}{\partial R} < 0$—see Proposition 1). As a result, investors are repaid in more states of the world, so a lower equilibrium deposit rate is still consistent with investor participation. A higher remuneration of CBDC improves the outside option of investors (at both the initial and interim dates), so the banker needs to offer a higher deposit rate to ensure investor participation.

The results so far highlight that the change in CBDC remuneration ω has two opposing effects on bank fragility θ^*. On the one hand, a higher remuneration is associated with a higher incentive to withdraw at date 1 and thus a large threshold θ^*. On the other hand, banks respond to the increase in remuneration by increasing deposit rates r_2^*, which leads to a reduction in bank fragility ceteris paribus. The overall effect of a change in ω on θ^* depends on which of this two effects dominates:

$$\frac{d\theta^*}{d\omega} = \frac{\partial\theta^*}{\partial\omega} + \frac{\partial\theta^*}{\partial r_2}\frac{dr_2}{d\omega}. \tag{6}$$

The next result offers some insight into the relative strength of these two forces.

Lemma 2. ***Elasticity of the failure threshold.*** *Let $\epsilon \equiv -\frac{r_2}{\theta^*}\frac{\partial\theta^*}{\partial r_2}$ denote the elasticity of the failure threshold with respect to the deposit rate. Higher CBDC remuneration reduces bank fragility, $\frac{d\theta^*}{d\omega} < 0$, if and only if $\epsilon > 1$.*

Proof. See Appendix C. □

Lemma 2 states that the indirect effect of higher CBDC remuneration dominates the direct effect whenever the failure threshold θ^* is very elastic to changes in the bank deposit rate r_2. In other words, higher CBDC remuneration needs to induce a sufficiently strong increase in deposit rates for overall fragility to fall. Figure 3 plots the range of parameters in (R, L) for which the elasticity is high enough for two levels of CBDC remuneration.

We now state our main result on CBDC remuneration and bank fragility.

Proposition 4. ***CBDC remuneration and bank fragility.*** *For a high investment return, $R \geq \overline{R}$, bank fragility θ^* is U-shaped in CBDC remuneration ω.*

Proof. See Appendix C. □

Figure 4 illustrates the case of high enough investment profitability R. It shows that a positive remuneration of CBDC can be desirable in the sense of max-

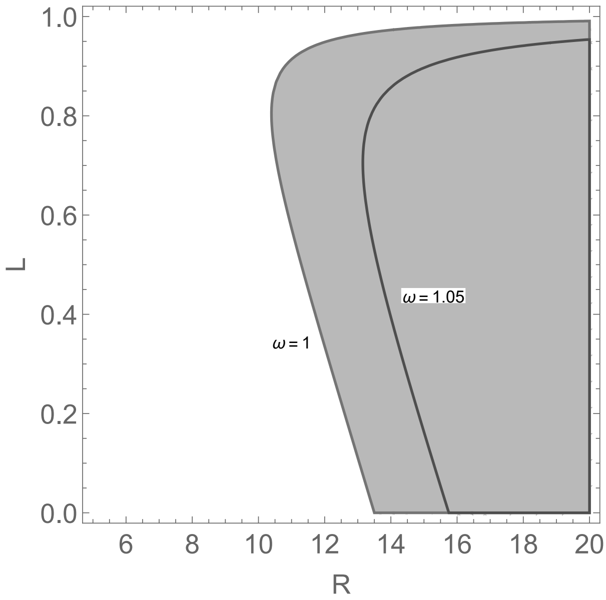


Figure 3: Elasticity of the failure threshold. The graph illustrate the range of values for the parameters R and L for which the elasticity of the failure threshold with respect to the deposit rate is $\epsilon > 1$, so the total derivative is negative, $\frac{d\theta^*}{d\omega} < 0$. This range corresponds to the grey area on the right of the two curves, which are drawn for two different values of ω. The light grey line is drawn for $\omega = 1$, while the black solid one is for $\omega = 1.05$.

imizing financial stability (minimizing bank fragility θ^*). The relationship between bank fragility and CBDC remuneration is U-shaped, with a unique minimum ω_{min}.

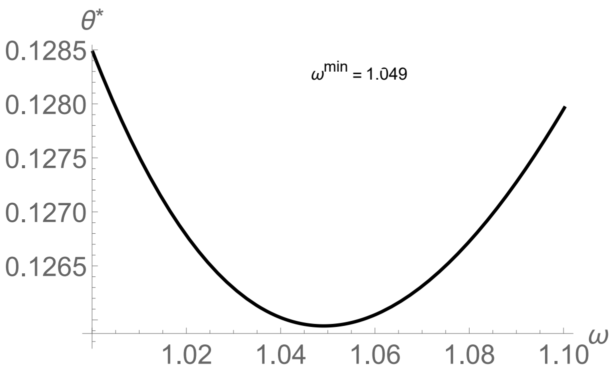


Figure 4: Bank failure threshold θ^* and CBDC remuneration. Minimum fragility is reached at roughly 4.9% interest on CBDC. Parameters: $L = 0.9$, $R = 15$.

4 Holding limits

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5 Extensions and discussion

We consider several extensions and robustness checks in this section.

5.1 Fundamental runs

In this subsection, we abstract from panic runs and limit attention to fundamental runs. This case can be studied by considering $L \to 1$, so the strategic complementarity among investors vanishes (note that $r_1^* = 1$ continues to hold).

The fundamental run threshold continues to be $\underline{\theta} = \frac{r_2}{R}$ (irrespective of CBDC remuneration ω). The banker's expected profit is $\Pi = \int_{\underline{\theta}}^{1} (R\theta - r_2) d\theta$ (which is also irrespective of CBDC remuneration ω), so $\frac{d\Pi}{dr_2} < 0$ and the banker chooses the lowest feasible level of r_2. This level is pinned down by the investors participation constraint:

$$\omega^2 \leq r_2(1 - \underline{\theta}) \equiv V_F. \tag{7}$$

Since the value of the deposit claim increases in r_2, $\frac{dV_F}{dr_2} > 0$, as long as $r_2 < \frac{R}{2}$, the participation constraint is binding in equilibrium. Solving for the smallest root yields

$$r_2^* = \frac{R}{2} - \sqrt{\left(\frac{R}{2}\right)^2 - R\omega^2}, \tag{8}$$

which confirms the supposition of an increasing deposit claim, $r_2^* < \frac{R}{2}$. We assume $R \geq 4\omega^2$ throughout. The equilibrium measure of bank stability is thus $\underline{\theta}^* = \frac{r_2^*}{R}$. Hence, more CBDC remuneration has the following impact on bank stability:

$$\frac{dr_2}{d\omega} = \frac{R\omega}{\sqrt{\frac{R^2}{4} - R\omega^2}} > 0. \tag{9}$$

The following proposition summarizes.

Proposition 5. ***Fundamental runs only.*** *Consider the case of $L \to 1$. Then, higher CBDC remuneration increases the return on bank deposits and thus the* *fundamental run threshold,* $\frac{dr_2}{d\omega} > 0$ *and* $\frac{d\theta}{d\omega} > 0$*, reducing bank stability.*

This analysis shows that panic runs are an essential ingredient of our main result (higher CBDC remuneration improves bank stability). When bank instability is limited to fundamental runs, higher CBDC remuneration always increases the long-term deposit rate and reduces the relevant measure of bank stability.

5.2 Bank risk-taking on the asset side

We have considered a fragile liability side of banks (uninsured deposits) as a source of financial instability so far. A large literature in banking is concerned with the risk-taking of banks on their asset side (e.g., risk choices and asset substitution). Our setup can be naturally extended along this dimension. Since the banker has to raise deposit rates to retain deposit funding in response to higher CBDC remuneration, it has lower skin in the game. This suggests that a moral hazard problem in risk choices would be more severe, contributing to financial instability via this additional channel. We plan to formally investigate this channel in future work.

5.3 Market power in the deposit market

In work in progress, we explore the role of market power in the deposit market for our results. We have so far assumed a monopolist bank, which can respond to the introduction of CBDC—a competition for its deposits—quite strongly by increasing deposit rates. However, when there are multiple banks competing for deposits, the ability of a given bank to attract or retain funding by raising rates may be more limited, suggesting a more prominent role of the direct effect of CBDC remuneration on bank fragility (resulting in lower financial stability). We seek to explore this issue in a spatial model of deposit competition in future work.

A Proof of Proposition 1

The proof builds on the arguments developed in Goldstein and Pauzner (2005) and on Carletti et al. (2022) who adapt these to the case of a profit-maximizing bank. The arguments in their proofs establish that, in the limit of $\epsilon \to 0$, there is a unique threshold value of fundamental, denoted as θ^*, below which all investors find it optimal to withdraw from the bank.

For $\theta \in (\underline{\theta}, \overline{\theta})$, a investor's decision to withdraw depends on what others do (i.e. their withdrawal choices). Suppose that all investors behave according to a threshold strategy s^*. Then, the fraction of investors withdrawing at date 1, $n(\theta, s^*)$, equals the probability of receiving a signal below s^* and is as follows:

$$n(\theta, s^*) = \begin{cases} 1 & \text{if } \theta \leq s^* - \epsilon, \\ \frac{s^* - \theta + \epsilon}{2\epsilon} & \text{if } s^* - \epsilon < \theta \leq s^* + \epsilon, \\ 0 & \text{if } \theta > s^* + \epsilon. \end{cases} \tag{10}$$

Investor withdrawal choices are characterized by the pair of thresholds $\{s^*, \theta^*\}$, which solve the following system of equations:

$$R\theta^* \left(1 - \frac{n(\theta^*, s^*) r_1}{L}\right) - (1 - n(\theta^*, s^*)) r_2 = 0, \tag{11}$$

and

$$r_2 Pr(\theta > \theta^* | s^*) = \omega r_1 Pr(\theta > \theta_n | s^*), \tag{12}$$

where $\theta_n = s^* + \epsilon - 2\epsilon \frac{L}{r_1}$ being the solution to $n(\theta, s^*)\, r_1 = L$ (illiquidity threshold).

Condition (11) identifies the level of fundamentals θ at which the bank is just able to repay the promised repayment to non-withdrawing investors (solvency threshold). Hence, it pins down the cutoff θ^*. Condition (12), instead, states that at the signal threshold s^* a investor is indifferent between withdrawing at date 1 and waiting until date 2, since the expected payoff at date 2, as captured by the

LHS in (12), is equal to the expected date 1 payoff, which is captured by the RHS in (12). Hence, given θ^* from (11), it pins down the threshold signal s^*.

Differentiating the LHS of (11) with respect to θ, we obtain

$$R\left(1 - \frac{n(\theta, s^*)r_1}{L}\right) - \frac{\partial n(\theta, s^*)}{\partial \theta}\left[R\theta\frac{r_1}{L} - r_2\right] > 0, \tag{13}$$

for any $\theta > \underline{\theta}$ since $r_1 > L$ and $\frac{\partial n(\theta,s^*)}{\partial \theta} \leq 0$. Taking the derivative of (11) with respect to $n(.)$, we obtain:

$$-R\theta\frac{r_1}{L} + r_2 < 0,$$

for any $\theta > \underline{\theta}$ since $r_1 > L$. Overall, this implies that the LHS in (11) strictly increases with θ and so it does the LHS in (12). Furthermore, rearranging (11) as follows:

$$R\theta^* - r_2 - n(\theta^*, s^*)\left[R\theta^*\frac{r_1}{L} - r_2\right] = 0,$$

it follows immediately that (11) is negative when evaluated at $\theta = \underline{\theta}$ and positive when $\theta = \overline{\theta}$. Using (12), this means that when $\theta = \underline{\theta}$, a investor expects to receive 0 when waiting a so strictly prefers to run. Symmetrically, when $\theta = \overline{\theta}$ so that the LHS in (11) is strictly above zero, a investor expects to receive $r_2 > \omega\, r_1$ when waiting until date 2. Since ωr_1 is larger than the RHS in (12), it follows that when $\theta = \overline{\theta}$, a investor strictly prefer not to run.

Overall, the analysis above also implies that $\underline{\theta} < \theta^* < \overline{\theta}$ and analogously that the threshold signal s^* falls within the range $\left(\underline{\theta} + \epsilon, \overline{\theta} - \epsilon\right)$. Given that $\underline{\theta} > 0$ and $\overline{\theta} \to 1$, it follows that the equilibrium pair $\{\theta^*, s^*\}$ falls in the range $(0, 1)$.

To obtain a closed-form expression, we perform a change of variable using (10) from which we obtain $\theta(n) = s^* + \epsilon(1 - 2n)$. At the limit, when $\epsilon \to 0$, $\theta(n) = s^*$, which identifies the run threshold and it is equal to the solution to

$$\int_0^{\widehat{n}(\theta^*)} r_2 dn = \int_0^{\overline{n}} \omega r_1\, dn \Rightarrow \widehat{n}\left(\theta^*\right) r_2 = \omega L. \tag{14}$$

Rewriting and solving for θ^* yields the expression stated in the proposition. Because of $L < 1 \leq r_1$, we have $\theta^* > \underline{\theta}$.

We turn to the comparative statics of bank fragility θ^* with respect to deposit rates r_1 and r_2 as well as CBDC remuneration ω and liquidation value L as well as a cross-partial derivative useful in the subsequent analysis:

$$
\begin{aligned}
\frac{\partial \theta^*}{\partial r_1} &= \frac{\omega \theta^*}{(r_2 - r_1\omega)} > 0, && (15)\\
\frac{\partial \theta^*}{\partial r_2} &= \frac{1}{R}\frac{r_2 - \omega L}{r_2 - r_1\omega} - \frac{\underline{\theta}\omega(r_1 - L)}{(r_2 - r_1\omega)^2} = \frac{r_2^2 - 2\omega r_1 r_2 + \omega^2 L r_1}{R(r_2 - r_1\omega)^2}, && (16)\\
\frac{\partial \theta^*}{\partial \omega} &= \underline{\theta}\frac{r_2(r_1 - L)}{(r_2 - r_1\omega)^2} > 0, && (17)\\
\frac{\partial \theta^*}{\partial L} &= -\omega\frac{\underline{\theta}}{r_2 - r_1\omega} < 0, && (18)\\
\frac{\partial \theta^*}{\partial R} &= -\frac{\theta^*}{R} < 0, && (19)\\
\frac{\partial^2 \theta^*}{\partial r_2 \partial L} &= \frac{\omega}{R}\frac{r_1\omega}{(r_2 - r_1\omega)^2} > 0, && (20)
\end{aligned}
$$

where we used $r_1 > L$ and $r_2 > \omega r_1$. Consider now the effect of r_2 on θ^*. The denominator is positive and the numerator is ambiguous. In particular, $\frac{d\theta^*}{dr_2} < 0$ whenever $r_2^2 - 2\omega r_1 r_2 + \omega^2 L r_1 < 0$. The roots of the quadratic equation are

$$
r_2^{A/B} = \omega r_1 \left(1 \pm \sqrt{1 - \frac{L}{r_1}}\right). \tag{21}
$$

The smaller root r_2^A is inadmissible as it implies $r_2 < \omega r_1$, a contradiction. Thus, only the bigger root $r_2^B > \omega r_1$ is admissible. Since this value is the maximum of the relevant deposit rates considered by the bank, as we will show shortly, we label it $r_2^{max} \equiv r_2^B$. To summarize, $\frac{\partial \theta^*}{\partial r_2} < 0$ whenever $r_2 < r_2^{max}$.

B Proof of Proposition 3

Consider the bank's problem stated in the main text. First, we report some intermediate results. Consider a bound on the investment return: $R > \underline{R} \equiv \frac{\omega\left(1+\sqrt{1-L}\right)^3}{1+\sqrt{1-L}-\omega}$. Note that this bound can also be written as $L > \underline{L}$. Assuming this bound ensures that the participation constraint of investors is slack at $r_2 = r_2^{max}$, that is $V(r_2^{max}) > \omega^2$. Note that $\theta^*(r_2^{max}) = \frac{\omega}{R}\left(1+\sqrt{1-L}\right)^2$ and $V(r_2^{max}) = \omega\left(1+\sqrt{1-L}\right) - \frac{\omega^2}{R}\left(1+\sqrt{1-L}\right)^3$, resulting in the stated bound $\underline{R}$. As a corollary, the bank does not always fail, $\theta^*(r_2^{max}) < 1$, making the bank's problem economically interesting. Note that $\frac{\partial \Pi}{\partial r_2} = -(1-\theta^*) - (R\theta^* - r_2)\frac{\partial \theta^*}{\partial r_2}$, so we have $\frac{\partial \Pi}{\partial r_2} < 0$ for all $r_2 \geq r_2^{max}$. As a result, the bound on R also ensures that the banker's choice of deposit rate is $r_2^* < r_2^{max}$.

We can further narrow down the deposit contracts that a profit-maximizing bank may choose to offer by noticing that the bank does not choose a deposit contract that entails $\theta^* = 1$. If a crisis is certain, the bank is certain to make zero profits. As a result, the bank chooses $r_2 > r_2^{min}$ where $\theta^*(r_2^{min}) \equiv 1$. Thus,

$$r_2^{min} \quad = \quad \frac{R+\omega L}{2} - \sqrt{\left(\frac{R+\omega L}{2}\right)^2 - R\omega}. \tag{22}$$

Note that $r_2^{min} < r_2^{max}$.

In the second step, we can write the Lagrangian as follows, where λ is the multiplier on the participation constraint of investors:

$$\mathcal{L} = \int_{\theta^*}^{1} (R\theta - r_2)d\theta - \lambda\left[\omega^2 - \int_{\theta^*}^{1} r_2 d\theta\right].$$

The Kuhn-Tucker conditions are

$$\begin{gathered} \lambda \geq 0, \quad \lambda\left[\omega^2 - \textstyle\int_{\theta^*}^{1} r_2 d\theta\right] = 0, \\ -\frac{\partial \theta^*}{\partial r_2}(R\theta^* - r_2) - \textstyle\int_{\theta^*}^{1} d\theta - \lambda\frac{\partial \theta^*}{\partial r_2} r_2 + \lambda(1-\theta^*) = 0. \end{gathered} \tag{23}$$

The third step is to determine whether the participation constraint of investors or zero marginal bank profits determine the equilibrium deposit rate r_2^*.

Consider a case in which $\partial\theta^*/\partial r_2 = 0$. In this case, we have $\lambda^* = 1$. That is, if such solution exists, then the participation constraint must bind. This is intuitive, as a bank would never choose such a deposit rate so high that this derivative is zero. It would stop a bit earlier. Our earlier analysis yielded that $r_2 = r_2^{max} = \omega(1+\sqrt{1-L})$ at $\partial\theta^*/\partial r_2 = 0$. Hence, $\theta^*(r_2^{max}) = \frac{\omega}{R}(1+\sqrt{1-L})^2$. Since the participation constraint is binding (as $\lambda^* > 0$), we get the knife-edge condition

$$1+\sqrt{1-L} = \frac{\omega R}{R-\omega(1+\sqrt{1-L})^2}, \tag{24}$$

which uniquely defines a bound $\underline{R}$ (stated above), $\overline{\omega}$, or $\underline{L}$ (implicitly given).

We use a continuity argument to generalize based on the knife-edge condition. That is, for a sufficiently small perturbation of the parameters, λ^* remains strictly positive (i.e., the participation constraint binds). That is, the participation constraint is binding for any L close enough to $\underline{L}$. We have for such L that $\frac{\mathrm{d}r_2^*}{\mathrm{d}\omega} = \frac{\omega^2(r_2^*)^2(1-L)}{(1-\theta^*)^2(r_2^*-\omega)^2} > 0$, $\frac{\partial^2\theta^*}{\partial r_2^2} = \frac{2}{R(r_2^*-\omega)}$, and $\frac{\mathrm{d}r_2^*}{\mathrm{d}L} = -\frac{\omega^3\cdot r_2^*}{(1-\theta^*)^2(r_2^*-\omega)R} < 0$. Thus, we can conclude by continuity that $\partial\theta^*/\partial r_2^* < 0$ for any $L > \underline{L}$ close enough to $\underline{L}$. It follows directly that for any $L > \underline{L}$ close enough to $\underline{L}$ the participation constraint is binding and the indirect effect

$$\frac{\partial\theta^*(1, r_2^*; \omega)}{\partial r_2}\frac{\mathrm{d}r_2^*}{\mathrm{d}\omega} < 0. \tag{25}$$

In sum, we have provided a cases for which the participation constraint binds in equilibrium. The numerical examples in the main text suggest that it does so for a much wider range of parameters.

B.1 Equilibrium deposit rate and its properties

Having established that the deposit rate r_2^* corresponds to the solution to the binding participation constraint for some $L > \underline{L}$, we now move on to prove the existence and uniqueness of r_2^*. Denote as $g(r_2, \omega)$ the depositors' net participation constraint:

$$g(r_2, \omega) = \omega^2 - \int_{\theta^*}^{1} r_2 d\theta. \tag{26}$$

The equilibrium deposit rate r_2^* is then given by the solution to $g(r_2, \omega) = 0$. Evaluating $g(r_2, \omega)$ at $r_2 = r_2^{min}$ and $r_2 = r_2^{max}$, we obtain $g(r_2^{min}, \omega) = -\omega^2 < 0$ and $g(r_2^{max}, \omega) = V(r_2^B) - \omega^2 > 0$, respectively, based on the definitions of r_2^{min}, r_2^{max}, and Assumption 1 on the lower bound on R. Differentiating $g(r_2, \omega)$ with respect to r_2, we obtain

$$\frac{\partial \theta^*}{\partial r_2} r_2 - \int_{\theta^*}^{1} d\theta < 0. \tag{27}$$

Taken together the monotonicity of $g(r_2, \omega)$ in r_2, $g(r_2^{mim}, \omega) < 0$ and $g(r_2^{max}, \omega) > 0$ imply that a solution for $g(r_2, \omega) = 0$ exists in the relevant parameter space and this solution is unique.

To proceed with the proof, we can now move on to study how r_2 changes with ω. To do this, we use the implicit function theorem, as follows:

$$\frac{dr_2}{d\omega} = -\frac{\frac{\partial g(r_2,\omega)}{\partial \omega}}{\frac{\partial g(r_2,\omega)}{\partial r_2}},$$

The denominator is positive, as shown in (27). Hence, the sign of $\frac{dr_2}{d\omega}$ is equal to the sign of the numerator, which is equal to

$$\frac{\partial g(r_2, \omega)}{\partial \omega} = 2\omega + \frac{\partial \theta^*}{\partial \omega} r_2 > 0. \tag{28}$$

It follows that r_2 monotonically increases with CBDC remuneration ω and formally

$$\frac{dr_2}{d\omega} = -\frac{2\omega + \frac{\partial \theta^*}{\partial \omega} r_2}{\frac{\partial \theta^*}{\partial r_2} r_2 - \int_{\theta^*}^{1} d\theta} > 0, \tag{29}$$

Finally, we derive the comparative statics of the deposit rate with respect to investment characteristics. Using the implicit function theorem again, the results $\frac{dr_2^*}{dL} < 0$ and $\frac{dr_2^*}{dR} < 0$ follow from $\frac{\partial g}{\partial L} = r_2 \frac{\partial \theta^*}{\partial L} < 0$ and $\frac{\partial g}{\partial R} = r_2 \frac{\partial \theta^*}{\partial R} < 0$.

C Proof of Lemma 2 and Proposition 4

Lemma. Using the expression for $\frac{dr_2}{d\omega}$ in Equation (29), we expand the expression for $\frac{d\theta^*}{d\omega}$:

$$\frac{d\theta^*}{d\omega} = \frac{\partial\theta^*}{\partial\omega} + \frac{\partial\theta^*}{\partial r_2}\frac{dr_2^*}{d\omega} \tag{30}$$

$$= \frac{\partial\theta^*}{\partial\omega} + \frac{\partial\theta^*}{\partial r_2}\frac{2\omega + \frac{\partial\theta^*}{\partial\omega} r_2^*}{1 - \theta^* - r_2^* \frac{\partial\theta^*}{\partial r_2}}. \tag{31}$$

Since the denominator is positive in the previous equation is positive, we get $\frac{d\theta^*}{d\omega} < 0$ whenever $\frac{\partial\theta^*}{\partial\omega}\left(1 - \theta^* - r_2^*\frac{\partial\theta^*}{\partial r_2}\right) + \frac{\partial\theta^*}{\partial r_2}\left(2\omega + \frac{\partial\theta^*}{\partial\omega} r_2^*\right)$, which simplifies to

$$\frac{\partial\theta^*}{\partial\omega}(1-\theta^*) + 2\omega\frac{\partial\theta^*}{\partial r_2} < 0. \tag{32}$$

Using the equilibrium deposit rate to replace $1 - \theta^* = \frac{\omega^2}{r_2^*}$ and the fact that $\frac{\partial\theta^*}{\partial r_2} = \frac{1}{r_2}\left[\theta^* - \omega\frac{\partial\theta^*}{\partial\omega}\right]$, we can re-express this condition as:

$$\theta^* + r_2^*\frac{\partial\theta^*}{\partial r_2} < 0, \tag{33}$$

which has the intuitive interpretation of an elasticity. In particular, the elasticity of the failure threshold with respect to deposit rate has to be bigger than 1 in absolute terms for the indirect effect to dominate and thus $\frac{d\theta^*}{d\omega} < 0$.

Rewriting condition (32) and using $1 - \theta^* = \frac{\omega^2}{r_2^*}$ yields $\omega\frac{\partial\theta^*}{\partial\omega} + 2r_2\frac{\partial\theta^*}{\partial r_2} < 0$. Using the expressions for the partial derivatives and mutiplying by the common denominator $R(r_2-1)^2$, we get $\omega r_2^2(1-L) + 2r_2(r_2^2 - 2\omega r_2 + \omega^2 L) < 0$. Rewriting and dividing by r_2 yields the following quadratic equation:

$$h(r_2, \omega) \equiv (r_2^*)^2 - \frac{3+L}{2}\omega r_2^* + \omega^2 L < 0. \tag{34}$$

If this condition holds, then $\frac{d\theta^*}{d\omega} < 0$ in equilibrium, where we used the equilibrium deposit rate r_2^*, which solves $g(r_2^*) = 0$.

Proposition. First, we determine whether $\frac{d\theta^*}{d\omega} < 0$ when evaluated at $\omega = 1$ is possible. Using condition (34), this boils down to $(r_2^*)^2 - \frac{3+L}{2} r_2^* + L < 0$. Consider next the limit of $R \to \infty$. Note that $\theta^* \to 0$ and thus $r_2^* \to 1$ for a given $L < 1$. Hence, the previous condition becomes $-\frac{1-L}{2} < 0$, which always holds. Thus, by continuity, there exists a $\overline{R}$ s.t. $\frac{d\theta^*}{d\omega} < 0$ at $\omega = 1$ for all $R \geq \overline{R}$.

Second, we show that $\frac{d\theta^*}{d\omega} > 0$ when ω is large. To do so, recall that we established that $\frac{dr_2^*}{d\omega} > 0$ and $r_2^* < r_2^{max}$. Then, we can denote ω^{max} such that $r_2^* \to r_2^{max}$ when $\omega \to \omega^{max}$. At this limit, Condition (33) is always violated since $\frac{\partial \theta^*}{\partial r_2} \to 0$ when $r_2 \to r_2^{max}$. Thus, $\frac{d\theta^*}{d\omega} > 0$.

Taken together, we have $\frac{d\theta^*}{d\omega} > 0\big|_{\omega=1} < 0$ and $\frac{d\theta^*}{d\omega}\big|_{\omega^{max}} > 0$, which implies that, when $R \geq \overline{R}$, there is at least a value of ω—denoted as ω^{min}—at which θ^* is minimized. To complete the proof, we need to show in a third step that ω^{min} is unique. The value ω^{min} solves $h(r_2^*, \omega_{min}) = 0$, where $h(r_2, \omega)$ is given in (34). Since r_2^* is a function of ω, $h(r_2(\omega), \omega)$ is a polynomial where ω is the main variable. The degree of the polynomial determines the number of possible values ω^{min}. Since $\frac{d\theta^*}{d\omega}\big|_{\omega=1} < 0$ and $\frac{d\theta^*}{d\omega}\big|_{\omega^{max}} > 0$, the number of solutions ω^{min} must be an odd number.

To determine the degree of the polynomial $h(r_2(\omega), \omega)$, it is useful to characterize a closed-form solution for r_2^*. Recall that r_2^* solves $g(r_2^*, \omega) = 0$, as characterized in (26). Substituting the expression for θ^* from (5), we obtain:

$$r_2^3 - r_2^2(R + \omega L) + r_2 R\omega(\omega + 1) - R\omega^3 = 0. \tag{35}$$

The equation (35) has three roots, which we can obtain solving the corresponding depressed cubic equation

$$y^3 + py + q = 0 \tag{36}$$

where $y = r_2 - \frac{R+\omega L}{3}$, $p = \frac{3R\omega(1+\omega) - (R+\omega L)^2}{3}$ and $q = \frac{-2(R+\omega L)^3 + 9(R+\omega L)R\omega(\omega+1) - 27R\omega^3}{27}$.

We focus on parameters such that $4p^3 + 27q^2 > 0$ (for which $p \geq 0$ is sufficient). As a result, there is only one real root, which is equal to:

$$y = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}. \tag{37}$$

The expression pinning down y and, in turn, r_2^* is a function of ω. One can show that ω only appears at a power of 1. This implies that $h(r_2(\omega), \omega)$ has at most two roots, of which only one can be in the range $1 < \omega < \omega^{max}$. Since the derivative is initially negative and eventually positive, there must be an odd number of crossings with zero within $[1, \omega_{max}]$. Hence, ω_{min} is unique.

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